Math 397: Exam 3 **Name:** ____________________

Summer Session II – 2017

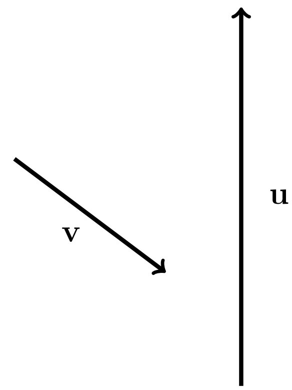
08/10/2017

145 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 22 pages (including this cover page) and 12 questions. Check that you have every page of the exam. ***You are to select any 10 of the first 12 problems, not including two bonus problems. If you attempt more than 10 of the first 12 problems, indicate clearly which 10 you wish to have graded.*** Answer these chosen questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page – being sure to indicate the problem number.

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
9	0	
10	0	
11	0	
12	0	
Total:	0	

1. (a) Given the vectors $\mathbf{u}$ and $\mathbf{v}$ below, sketch $\mathbf{u}+\mathbf{v}$ and $\mathbf{v}-\mathbf{u}$.



Let $\mathbf{a} = 2\,\hat{\mathbf{\imath}} - \hat{\mathbf{\jmath}} + 3\,\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{\imath}} - \hat{\mathbf{k}}$.

(b) Find $2\mathbf{b} - \mathbf{a}$.

(c) Find a unit vector in the direction of $\mathbf{a}$.

(d) Find the angle between $\mathbf{a}$ and $\mathbf{b}$.

(e) Are $\mathbf{a}$ and $\mathbf{b}$ orthogonal? Justify your answer.

2. Let $\mathbf{a} = 2\,\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{k}}$.

   (a) Find a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

   (b) Find the area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$.

   (c) Find the volume of the parallelepiped generated by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c} = \langle 1, -1, 4 \rangle$.

   (d) Are $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ coplanar?

3. Let $P(4, -6, 4)$, $Q(7, -5, 3)$, and $R(8, -3, 2)$ be points in $\mathbb{R}^3$.

   (a) Find the equation of the line through Q and R.

   (b) Find the equation of the plane perpendicular to the line from (a) and containing the point P.

   (c) Find the distance from the point P to the line from (a).

4. Let $f(x, y, z) = \dfrac{2x + ye^{3y} - z}{x}$.

   (a) Find the equation of the tangent plane to $f(x, y, z)$ at the point $(1, 0, -1)$.

   (b) Find f_{xy}.

   (c) Find $\dfrac{\partial^2 f}{\partial y \partial z}$.

5. Let $f(x, y, z) = e^{x^2+y^2+z^2}$.

   (a) Find the rate of change of $f(x, y, z)$ in the direction of $\mathbf{u} = -\hat{\mathbf{\imath}} + \hat{\mathbf{\jmath}} - \hat{\mathbf{k}}$ at the point $(0, 2, -1)$.

   (b) What is the direction of maximum increase for the function $f(x, y, z)$ at the point $(0, 2, -1)$? What is the direction of maximum decrease for the function $f(x, y, z)$ at the point $(0, 2, -1)$?

   (c) Find the rate of change for $f(x, y, z)$ at the directions you found in (b).

6. Choose either (a) or (b) to complete. *You **do not** need to do both. Choose only of of the two and complete it.*

   (a) Find and classify the critical points for the function $f(x, y, z) = e^z(z^2 - y^2 - 2x^2)$.

   (b) Find the maximum and minimum values of $F(x, y, z) = 2x^3 + y^3 + 2z^{3/2}$ if x, y, and z satisfy $x^4 + y^4 + z^2 = 33$ and $xyz \neq 0$. [Note: $xyz \neq 0$ simply says that none of x, y, or z are zero.]

7. Complete the following parts:

   (a) Compute the following integral:

$$\int_0^3 \int_0^{9-x^2} \frac{xe^{3y}}{9-y}\, dy\, dx$$

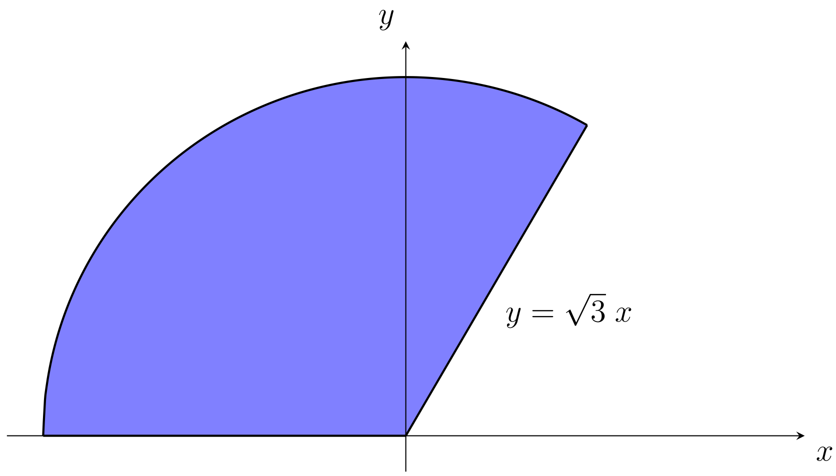
(b) Set up completely as possible *but do not integrate* any integral which would compute the volume of the region enclosed by $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$.

8. Complete the following parts:

   (a) Change the integral given below into an integral in polar coordinates. Be sure to set up the integral completely as possible *but do not evaluate the integral*.

$$\iint_R \cos(x^2 + y^2)\, dA$$

   The region R is the shaded region portion of the unit circle in the figure below:



(b) Change the integral given below into an integral in cylindrical coordinates. Be sure to set up the integral completely as possible *but do not evaluate the integral*.

$$\iiint_R \left(2+\sqrt{x^2+y^2}\right)\, dV$$

The region R is given by $R = \left\{(x,y,z)\colon \sqrt{x^2+y^2} \le \frac{z}{2} \le 3\right\}$.

(c) Change the integral given below into an integral in spherical coordinates. Be sure to set up the integral completely as possible *but do not evaluate the integral*.

$$\iiint_R \frac{y}{\sqrt{x^2+y^2+z^2}}\, dV$$

The region R is the collection of points between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$, where $y>0$ and $z>0$.

9. Let C be the directed line segment from the point $(-2, 1)$ to the point $(1, 3)$.

(a) Compute $\int_C (x + 2y)\, dx$.

(b) Compute $\int_C (x + 2y)\, dy$.

(c) Compute $\int_C (x + 2y)\, ds$

10. Complete the following parts:

(a) Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$ and $\mathbf{r}(t) = (t, 3t^2, 2t^3)$ for $0 \le t \le 1$.

(b) Compute

$$\oint_C -y\ dx + x\ dy$$

where C is the circle of radius 3 centered at the origin, oriented counterclockwise.

11. Let $\mathbf{F} = (2xy + 1)\,\hat{\mathbf{i}} + (x^2 - 1)\,\hat{\mathbf{j}}$.

(a) Compute div $\mathbf{F}$.

(b) Use the curl to show that $\mathbf{F}$ is conservative.

(c) Find a potential function for $\mathbf{F}$.

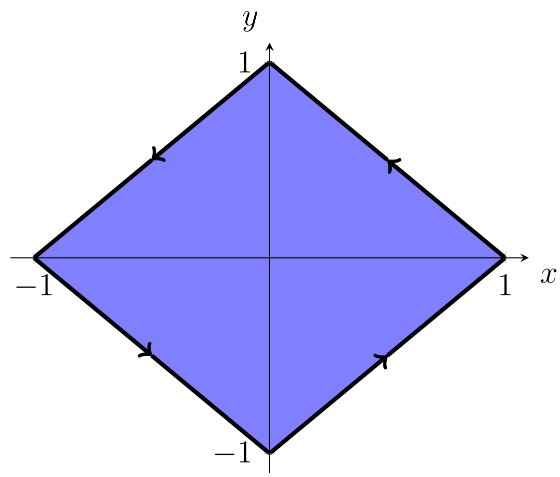
(d) Compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C : [0,1] \to \mathbb{R}^2$ is the path given by

$$\mathbf{r}(t) = \left(e^{t^2-t} + \sin\left(\pi \cos\left(\frac{\pi t}{2}\right)\right) + 2t,\ \frac{1}{t^2+2t-4} - \sin(\pi t) + \frac{1-t}{4} \right)$$

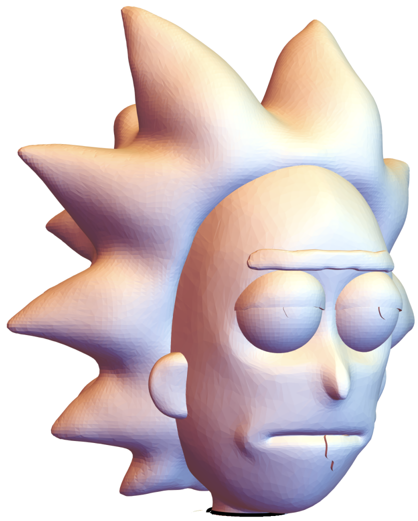
12. Compute

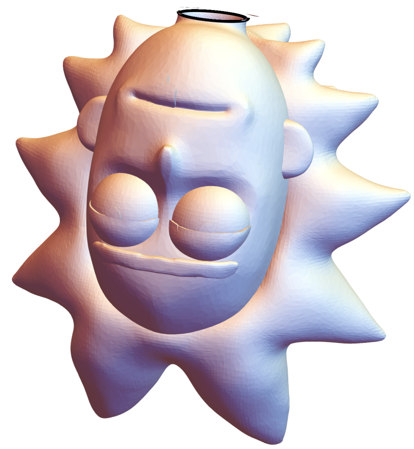
$$\oint_C (xy^2 + 2y^3 + y)\ dx + (x^2y + 6xy^2 + 10x)\ dy$$

where C is the boundary of the square, oriented counterclockwise, shown below



Bonus 1: Suppose S is the surface consisting of 214,252 polygons that are smoothly connected to form the visage of the scientist Rick Sanchez—also known as Pickle Rick, where the equation of the base of the neck (highlighted on the surface in dark black) is the circle $4x^2 + 4y^2 = 9$ in the plane $z = 47$ (see the figures below).[1]





If $\mathbf{F} = (x - 2y)\,\hat{\mathbf{\imath}} + (2x - y)\,\hat{\mathbf{\jmath}} + (xz + yz - x\sin(yz))\,\hat{\mathbf{k}}$, calculate

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$$

Be sure to justify all steps in your calculations and any theorems used thoroughly.

[1] ChaosCoreTech. 2017, July. Rick Sanchez [Rick and Morty]. https://pinshape.com/items/33235-3d-printed-rick-sanchez-rick-and-morty.
Rick and Morty. Warner Bros. Television. Warner Bros. Television Distribution. July 2017. Television.

Bonus 2: Verify the Divergence Theorem for the vector field $\mathbf{F} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$ over the sphere S of radius R centered at the origin, i.e. $x^2 + y^2 + z^2 = R^2$.