Name: Caleb McWhorter — Solutions **Fall 2014**
MAT 221: Section 1 **Quiz 6**

Note: *You must show the details of the work to receive credit. Simply providing the final answer [from a calculator] will get* ***ZERO*** *points.*

Formulae:

(i) If the sample is from a normal distribution, then the sampling distribution of $\overline{X}_n$ is normal $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ regardless of the sample size.

(ii) If n is large ($n \geq 30$), according to the Central Limit Theorem (CLT), the sampling distribution of $\overline{X}_n$ is *approximately* normal $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ regardless of the population distribution.

1. The measured glucose level, in mg/dl, one hour after having a sugary drink has a normal distribution with mean 125 and standard deviation 14.

(a) (3 points) If a single glucose measurement is made, what is the probability that the glucose level in that measurement is above 132?

$$z_{132} = \frac{132 - 125}{14} = \frac{7}{14} = 0.50 \rightsquigarrow 0.6915$$

Therefore, $P(\geq 132) = 1 - 0.6915 = 0.3085.$

(b) (3 points) If a random sample of 16 glucose measurements is taken, what is the probability that the average of those 16 measurements is above 132?

$$z_{132} = \frac{132 - 125}{\frac{14}{\sqrt{16}}} = \frac{7}{\frac{14}{4}} = \frac{7}{3.5} = 2.00 \rightsquigarrow 0.9772$$

Therefore, $P(\geq 132) = 1 - 0.9772 = 0.0228.$

2. The length of time a particular brand of battery lasts (called the lifetime of the battery) has population mean $\mu = 65$ days and population standard deviation $\sigma = 20$ days.

(a) (3 points) If a random sample of 60 batteries of that brand is taken, what the probability that the sample average lifetime is at most 72 days.

$$z_{72} = \frac{72 - 65}{\frac{20}{\sqrt{60}}} = \frac{7}{\frac{20}{7.74597}} = \frac{7}{2.58199} = 2.71 \rightsquigarrow 0.9966$$

(b) (1 point) If the sample size was 15 (instead of 60), could we have done the computation in part a? Explain.

No. We needed the Central Limit Theorem for the group z-score calculation in the previous part. We can only use this if the original population is normally distributed or if n is sufficiently large. The population is not known to be normal and this n is not sufficiently large (≥ 30).