

**Note:** You must show the details of the work to receive credit. Simply providing the final answer [from a calculator] will get **ZERO** points.

**Formulae:** Sample count  $X$  out of a simple random sample (SRS) of size  $n$ , where the population proportion is  $p$ , has a Binomial distribution with parameters  $n$  and  $p$ .

i.) If  $np \geq 10$  and  $n(1 - p) \geq 10$ , then  $X$  is approximately normal,  $N(\mu = np, \sigma = \sqrt{np(1 - p)})$ .

ii.) If  $np \geq 10$  and  $n(1 - p) \geq 10$  then  $\hat{p} = \frac{x}{n}$  is approximately normal,  $N\left(\mu = p, \sigma = \sqrt{\frac{p(1 - p)}{n}}\right)$ .

1. According to the Gallup-Healthways Well-Being Index<sup>1</sup>, “9% of Americans are ‘stressed’ .”

(a) (2 points) If a simple random sample of 4 Americans is taken, what is the probability that 2 or more of them in the sample are “stressed”?

$$P(2) + P(3) + P(4) = 0.0402 + 0.0027 + 0.0001 = 0.043$$

(b) (4 points) If a simple random sample of 200 Americans is taken, what is the probability that at least 23 of them in the sample are “stressed”? [Use the Normal Approximation.]

Using the Normal approximation ( $np = 18$  and  $n(1 - p) = 182$ ), the distribution is approximately  $N(\mu = np, \sigma = \sqrt{np(1 - p)}) = N(18.0, 4.047)$ . Then

$$z_{23} = \frac{23 - 18}{4.047} = \frac{5}{4.047} = 1.235 \rightsquigarrow 0.8916$$

Therefore,  $P(\geq 23) = 1 - 0.8916 = 0.1084$ .

(c) (4 points) If a simple random sample of 240 Americans is taken, what is the probability that at most 13% the sample are “stressed”? [Use the Normal Approximation.]

Using the Normal approximation ( $np = 21.6$  and  $n(1 - p) = 218.4$ ), the distribution is approximately  $N\left(\mu = p, \sigma = \sqrt{\frac{p(1 - p)}{n}}\right) = N(0.09, 0.0185)$ . Then

$$z_{13} = \frac{0.13 - 0.09}{0.0185} = \frac{0.04}{0.0185} = 2.16 \rightsquigarrow 0.9847$$

<sup>1</sup><http://www.gallup.com/poll/106915/Gallup-Daily-US-Mood.aspx>