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MAT 221: Section 1 **Quiz 8**

Note: *You must show the details of the work to receive credit. Simply providing the final answer [from a calculator] will get **ZERO** points.*

Formulae: Suppose the population standard deviation σ is known and either the sample is large ($n \geq 30$) or the population is normal. Let $\overline{x}$ denote the sample mean from a sample population of size n.

(i) Then the confidence interval for the population mean μ is $\overline{x} \pm z^* \dfrac{\sigma}{\sqrt{n}}$, where $z^* = 1.645$ for 90% confidence interval, $z^* = 1.960$ for 95%, and $z^* = 2.576$ for 99%.

(ii) Sample size n needed to guarantee a margin of error of at most m (i.e., to estimate within $\pm m$) for the population mean μ when the population standard deviation σ is known is $n \geq \left(\dfrac{z^* \sigma}{m} \right)^2$.

1. A machine is used to fill soda bottles. The amount of soda dispensed into each bottle varies slightly and is known to have a normal distribution with population standard deviation $\sigma = 2.76$ ml.

(a) (5 points) A random sample of 25 bottles filled by the machine is taken and the amount of soda filled in each bottle was measured. From this sample data, the sample mean was calculated to be 578.8 ml. Find a 99% confidence interval for the population mean amount of soda filled by the machine.

For a 99% confidence interval, $z^* = 2.576$. *Then* $z^* \dfrac{\sigma}{\sqrt{n}} = 2.576 \dfrac{2.76}{\sqrt{25}} = 1.422$. *Therefore, the confidence interval is*

$$\left(\overline{x} - z^* \frac{\sigma}{\sqrt{n}} \ , \ \overline{x} + z^* \frac{\sigma}{\sqrt{n}} \right)$$
$$(578.8 - 1.422 \ , \ 578.8 + 1.422)$$
$$(577.378 \ , \ 580.222)$$

(b) (5 points) We would like to collect another sample such that we would be able to estimate the population mean amount of soda filled by the machine within ± 0.35 ml with 95% confidence. What minimum sample size should we take?

The error in the estimate is the term $z^* \dfrac{\sigma}{\sqrt{n}}$. *Using a 95% confidence interval, we have* $z^* = 1.960$. *Therefore,*

$$n \geq \left(\frac{z^* \sigma}{m} \right)^2 = \left(\frac{1.960 \cdot 2.76}{0.35} \right)^2 = \left(\frac{5.4096}{0.35} \right)^2 = 15.456^2 = 238.888 \rightsquigarrow 239$$

Therefore, the sample should use a minimum of 239 people.