Name: *Caleb McWhorter — Solutions* **MAT 221**
Fall 2014 **Problem Set 2**

Problem 1: Fill in the blank:

(a) The five number summary includes the following five measurements:

$minimum$, Q_1, $median$, Q_3, and $maximum$.

(b) The $median$ and $midrange$ are robust measures of center while $mean$ and $midrange$ are not.

(c) The standard deviation, σ, measures the spread about the $mean$.

(d) For a normal distribution, 68% of observations fall within one standard deviation of the mean, 95% within two standard deviations from the mean, and 99% within three standard deviations from the mean.

(e) Linear transformations do not change the $shape$ of a normal distribution but can change the $mean$ and $standard\ deviation$ of a distribution.

(f) If a class had a median exam grade of 61 and a mean exam grade of 66, the teacher may curve the class grade. Suppose the teacher adds 15 points to everyones grade. Then the new median grade is 76 and the new mean exam grade is 81.

(g) Generalizing (f), if one applies the linear transformation $ax + b$ to a data set with median m and mean $\overline{x}$, the new median is $am + b$ and the new mean is $a\,\overline{x} + b$.

Problem 2: Is the mean or standard deviation more sensitive to outliers? Give an example to explain.

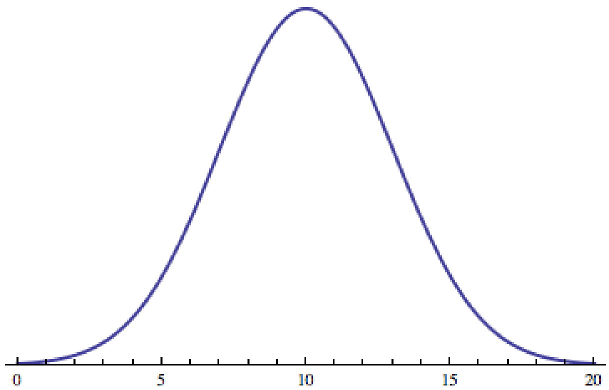
The standard deviation is more sensitive. Consider $0, 1, 2$*, which has mean 1 and standard deviation 1. The set* $0, 1, 2, 10$ *has mean 3.25 but standard deviation 4.57.*

Problem 3: Draw the stemplot (stem-and-leaf plot) for the following data set:

$$15 \quad 16 \quad 18 \quad 20 \quad 21 \quad 23 \quad 23 \quad 23 \quad 31 \quad 37 \quad 51 \quad 56 \quad 57$$

1	568
2	01333
3	17
4	
5	167

Problem 4: Look at the following normal distribution:



Using the graph, answer the following questions about the distribution:

(a) What is the median What is the mean?

The median and mean are both 10.

(b) What is the standard deviation?

The standard deviation is approximately the distance from 10 (the mean) to where the curve switches the direction it 'bends', i.e. at 13. So the standard deviation is approximately $13-10=3$.

(c) What percentage of data values fall between 7 and 10?

These are within 1 standard deviation of the mean, so it must be 68%.

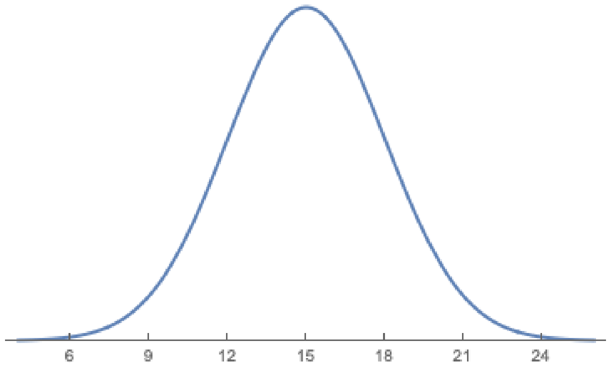
(d) What percentage fall between 10 and 16?

16 is two standard deviations above the mean. 95% of data falls within two standard deviations of the mean. But we only want the right side. So it must be $95/2 = 47.5\%$.

(e) What is the area under the curve between 7 and 10, and what is the area under the curve between 10 and 16?

The area is the corresponding proportion. So the areas are 0.68 *and* 0.475*, respectively.*

Problem 5: Sketch a normal curve with mean 15 and standard deviation 3. Mark the scores that are within $\pm 3\,\sigma$ from the mean.



Problem 6: Recall the z-score:

$$z = \frac{x - \mu}{\sigma}$$

Explain verbally what z is measuring. Why does this measure how 'unusual' a value is from a distribution?

The value $x - \mu$ measures how far x is from μ. Dividing this by σ measures 'how many σ fit into $x - \mu$', i.e. how many standard deviations x is from μ. If $z < 0$, then this is how many standard deviations to the left x is from μ, while if $z > 0$ this is how many standard deviations to the right x is from μ.

Problem 7: There are two major tests of readiness for college: the ACT and the SAT. ACT scores are reported on a scale from 1 to 36. This year, the ACT had mean 21.5 and standard deviation 5.4. The SAT scores are reported on a scale from 600 to 2400. This year, the SAT had mean 1498 and standard deviation 316. Suppose Jessica and Ashley took the SAT and ACT. Jessica took the SAT and scored 1825 while Ashley took the ACT and scored 28. Based on these scores, who do you think is better prepared for college? Explain your answer.

$$z_J = \frac{1825 - 1498}{316} = \frac{327}{316} = 1.0348$$
$$z_A = \frac{28 - 21.5}{5.4} = \frac{6.5}{5.4} = 1.2037$$

The z-score measures 'how unusual' a value is. The larger z is (in absolute value), the more 'unusual' the score. Therefore, Ashley's score is more unusual and is more prepared for college.