Name: *Caleb McWhorter — Solutions* **MAT 221**
Fall 2014 **Problem Set 5**

Problem 1: Suppose in a raffle, one can either win \$1, \$ 2, or \$5. Let X denote the amount of money you can get if you play the raffle. The probability distribution of X is given by

X	0	1	2	5
P(X)	*0.55*	0.30	0.10	0.05

(a) Find $P(X = 0)$ and fill it in on the table above.

$$P(X = 0) = 1 - 0.30 - 0.10 - 0.05 = 0.55$$

(b) Find the mean, μ_X, of the random variable X.

$$\mu_X = \sum XP(X) = 0(0.55) + 1(0.30) + 2(0.10) + 5(0.05) = 0.75$$

(c) Find the variance and standard deviation for the random variable X.

x	$x - \mu$	$(x-\mu)^2$	$(x-\mu)^2 P(x)$
0	-0.75	0.5625	0.3094
1	0.25	0.0625	0.0188
2	1.25	1.5625	0.1563
5	4.25	18.0625	0.9031
			Total: 1.3876

Therefore, $\sigma^2 = 1.3876$ *so that* $\sigma = \sqrt{1.3876} = 1.18$.

(d) Now suppose you have to pay \$1 to play the raffle. Let Y be the random variable that represents your net profit. Find μ_Y, the mean of Y. What is the standard deviation of Y?

We have $Y = X - 1$ *so that* $\mu_Y = \mu_X - 1 = 0.75 - 1 = -0.25$ *and* $\sigma_Y^2 = \sigma_X^2 = 1.3876$ *so that* $\sigma_Y = 1.18$.

Problem 2: Suppose you roll two dice and take the sum of the numbers you see. Let X denote the sum and $P(X)$ denote the probability of getting the sum X.

(a) For $X = 1, 2, \ldots, 12, 13$, find $P(X)$.

X	1	2	3	4	5	6	7	8	9	10	11	12	13
$P(X)$	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0

(b) Find $P(X \geq 10)$. Find $P(X \leq 10)$. What about $P(X < 10)$?

$$P(X \geq 10) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36} = 0.1667$$
$$P(X \leq 10) = 1 - P(11 \textit{ or } 12) = 1 - P(11) - P(12) = 1 - \frac{2}{36} - \frac{1}{36} = \frac{33}{36} = 0.9167$$
$$P(X < 10) = 1 - P(X \geq 10) = 1 - \frac{6}{36} = \frac{30}{36} = 0.8333$$

Problem 3: Suppose you have independent random variables X, Y with $\mu_X = 25$, $\sigma_X = 5$, $\mu_Y = 10$, and $\sigma_Y = 1$. Find the mean and standard deviation for the random variable Z if. . .

(a) $Z = 5X - 3$

$$\mu_Z = 5\mu_X - 3 = 5(25) - 3 = 122$$
$$\sigma_Z^2 = 5^2\sigma_X^2 = 25 \cdot 25 = 625 \Rightarrow \sigma_Z = 25$$

(b) $Z = 3Y - 2X$

$$\mu_Z = 3\mu_Y - 2\mu_X = 3(10) - 2(25) = -20$$
$$\sigma_Z^2 = 3^2\sigma_Y^2 + 2^2\sigma_X^2 = 9(1) + 4(25) = 109 \Rightarrow \sigma_Z = 10.44$$

(c) Suppose that X and Y were not independent. Instead, suppose they had correlation 0.20. Find the mean and standard deviation for the random variable Z for the two cases given in (b).

$$\mu_Z = 3\mu_Y - 2\mu_X = 3(10) - 2(25) = -20$$
$$\sigma_Z^2 = 3^2\sigma_Y^2 + 2^2\sigma_X^2 + 2\rho(3\sigma_Y)(2\sigma_X) = 3^2 \cdot 1^2 + 2^2 \cdot 5^2 + 2(0.20) \cdot (3 \cdot 1)(2 \cdot 5) = 50$$

so that $\sigma_Z = \sqrt{50} = 7.07$.