Name: _____*Caleb McWhorter — Solutions*_____   **MAT 221**
Fall 2014   **Problem Set 6**

Problem 1: Fill in the blanks:

(a.) If $P(A \text{ and } B) = P(A)P(B)$, then A and B are _____*independent*_____ events.

(b.) The _____*Law of Large Numbers*_____ says the mean of values of a random variable X observed after many trials must approach the mean μ_X.

(c.) The standard deviation is the square root of the _____*variance*_____.

(d.) If $P(A \text{ or } B) = P(A) + P(B)$, then A and B are _____*disjoint*_____ events.

(e.) The _____*sample space*_____ of a random phenomenon is the set of all possible outcomes.

(f.) The total area under a _____*probability distribution*_____ is 1. A common example of this is a

_____*normal distribution*_____.

(g.) The probability of a single value in a _____*continuous*_____ probability distribution is always 0.

(h.) For events A, B, $P(\underline{\ \ A \text{ or } B\ \ })$ is the probability that at least one of them occurs.

(i.) If a sample space has n events and all events have probability $1/n$ of occurring, then the

density curve is a _____*uniform*_____ distribution and each event is equally likely to occur.

Problem 2: The probability that a particle has velocity v km/s has a uniform distribution on the interval $[100, 300]$.

(a) Sketch the density curve.

This is a rectangle from $x = 100$ *to* $x = 300$ *with height* $y = \frac{1}{300-100} = \frac{1}{200}$.

(b) Compute the probability that the particle is moving at least 250 km/s.

$P(\geq 250) = (300 - 250) \cdot \frac{1}{200} = \frac{50}{200} = 0.25$

(c) Compute the probability that the particle is moving less than 150 km/s.

$P(\leq 150) = (150 - 100) \cdot \frac{1}{200} = \frac{50}{200} = 0.25$

(d) Compute the probability that the particle is moving between 150 km/s and 250 km/s

$P(between 150 \text{ and } 250) = (250 - 150) \cdot \frac{1}{200} = \frac{100}{200} = 0.5$

Problem 3: Consider the Titanic passenger mortality data given in the following table: Consider

	Men	Women	Boys	Girls	Total
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	45	2223

the following events:

S: A randomly chosen Titanic passenger survived.
D: A randomly chosen Titanic passenger died.
M: A randomly chosen Titanic passenger is a man.
W: A randomly chosen Titanic passenger is a woman.
B: A randomly chosen Titanic passenger is a boy.
G: A randomly chosen Titanic passenger is a girl.

(a.) Find $P(D)$. $P(D) = \dfrac{1517}{2223} = 0.6824$

(b.) Find $P(M \text{ and } D)$. $P(M \text{ and } D) = \dfrac{1360}{2223} = 0.6118$

(c.) Find $P(M \mid D)$. $P(M \mid D) = \dfrac{1360}{1517} = 0.8965$

(d.) Find $P(B \text{ or } G \mid S)$. $P(B \text{ or } G \mid S) = \dfrac{29 + 27}{706} = \dfrac{56}{706} = 0.0793$

Problem 4: A rare genetic disease is discovered and it is known that only 1 in 100 people has it. A test for the disease is developed. The test is positive for 99% of people who have the disease, while the test is also positive for 10% of people who do not have the disease.

(a) Construct a 2-step tree diagram for a randomly selected person and label the branches (including the final branch) with the appropriate probabilities.
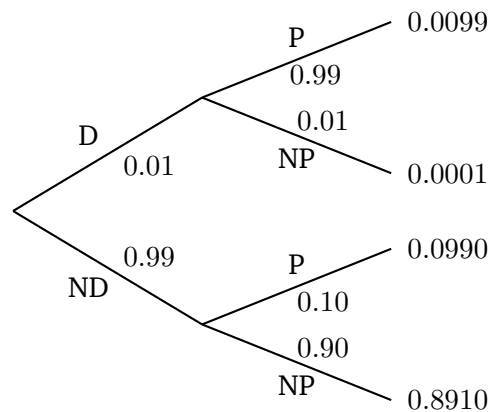
(b) What is the probability that a randomly selected person shows a positive test result?

$$P(\text{positive}) = 0.0099 + 0.0990 = 0.1089$$

Problem 5: The amount of beers consumed by a Syracuse University student is roughly approximated by a normal distribution with mean $\mu = 20$ and standard deviation $\sigma = 5$.

(a) What is the probability that a randomly selected student consumes more than 25 beers a week?

$$z_{25} = \frac{25 - 20}{5} = \frac{5}{5} = 1.0 \rightsquigarrow 0.8413 \Rightarrow 1 - 0.8413 = 0.1587$$

(b) What is the probability that a randomly selected student consumes less than 15 beers a week?

$$z_{15} = \frac{15 - 20}{5} = \frac{-5}{5} = -1.0 \rightsquigarrow 0.1587$$

(c) What is the probability that a randomly selected student consumes between 15 and 25 beers a week?

$$0.8413 - 0.1587 = 0.6826$$

Problem 6: A card is drawn from a standard 52-card deck, which consists of four suits (Spade, Club, Heart, Diamond) each with the following cards, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace. Spades and Clubs are black while Hearts and Diamonds are red. Consider the following events:
   A: The card is an Ace.
   B: The card is red (Heart or Diamond).
   C: The card is the Queen of Spade.
   D: the card is a face card (Jack, Queen, or King).

(a.) Find the following probabilities: $P(A)$, $P(B)$, $P(C)$, and $P(D)$.

$$P(A) = \frac{4}{52} \qquad P(B) = \frac{13 + 13}{52} = \frac{26}{52} \qquad P(C) = \frac{1}{52} \qquad P(D) = \frac{3 + 3 + 3 + 3}{52} = \frac{12}{52}$$

(b.) Find $P(A \text{ and } B)$ and $P(A \text{ or } B)$. Are A and B disjoint? Explain.

$P(A \text{ and } B) = \dfrac{2}{52}$ and $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \dfrac{4}{52} + \dfrac{26}{52} - \dfrac{2}{52} = \dfrac{28}{52}$.
No, A and B are not disjoint because $P(A \text{ and } B) \neq 0$.

(c.) Find $P(C \text{ and } D)$ and $P(C \mid D)$. Are C and D independent? Explain.

$P(C \text{ and } D) = \dfrac{1}{52}$ and $P(C \mid D) = \dfrac{1}{12}$. *No, C and D are not independent because*
$P(C)P(D) = \dfrac{1}{52} \cdot \dfrac{12}{52} = \dfrac{12}{2704} \neq P(C \text{ and } D)$.