Name: *Caleb McWhorter — Solutions* **MAT 221**
Fall 2014 **Problem Set 7**

Formulae:

(i) The sampling distribution for a large SRS ($n \geq 30$) of size n from a population with mean μ and standard deviation σ is $N(\mu, \sigma/\sqrt{n})$.

(ii) Binomial: The mean and standard deviation of a binomial count X and a sample proportion $\hat{p} = X/n$ are $\mu_X = np, \sigma_X = \sqrt{np(1-p)}, \mu_{\hat{p}} = p$, and $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$.

(iii) The Normal approximation to the binomial distribution $B(n,p)$,when n is large, is $X \approx N(np, \sqrt{np(1-p)})$ and $\hat{p} \approx N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$.

(iv) Binomial Coefficient: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

(v) Binomial Probability: If X has binomial distribution $B(n,p)$ then $P(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Problem 1: The amount of liquor consumed by a Syracuse University student during the weekend of a basketball game is normally distributed with mean 450 mL and standard deviation of 60 mL.

(a) What is the probability that a randomly selected student drinks more than 525 mL of liquor over the course of the weekend?

$$z_{525} = \frac{525-450}{60} = \frac{75}{60} = 1.25 \rightsquigarrow 0.8944 \Rightarrow 1 - 0.8944 = 0.1056$$

(b) What is the probability that a randomly selected *group* of 36 students drinks more than 525 mL of liquor over the course of the weekend?

$$z_{525} = \frac{525-450}{60/\sqrt{36}} = \frac{75}{10} = 7.5 \rightsquigarrow 1.00 \Rightarrow 1 - 1.00 = 0.0000$$

(c) If 10 students are selected at random, what is the probability that the average amount of liquor they consumed was less than 420 mL? What do you need to know to make this calculation? Explain.

$$z_{420} = \frac{420-450}{60/\sqrt{10}} = \frac{-30}{18.9737} = -1.58 \rightsquigarrow 0.0571$$

For this calculation, we need the Central Limit Theorem. But then we need $n \geq 30$ or the distribution to be normal. Because our n is not large, we needed that the liquor consumed is normally distributed.

Problem 2: UCONN considers any player making more than 20% of their free throw shots to be a "talented" player. A study of UCONN's basketball team over the last 50 years using APBRmetrics showed that only 20% of UCONN players are "talented" (using their metric). UCONN claims that their "talented" players make 30% of their free throws.

(a) If a "talented" player on the UCONN team gets a five chances during a game at a free throw shot, what is the probability that he makes at most 2 of these shots?

$$P(0) + P(1) + P(2) = 0.1681 + 0.3602 + 0.3087 = 0.8370$$

(b) What is the approximate probability that simple random sample of 50 UCONN players reveals that at least 13 of them are "talented?"

We have $N(np, \sqrt{np(1-p)}) = N(10, 2.828)$. *Note* $np = 10$ *and* $n(1-p) = 40$.

$$z_{13} = \frac{13 - 10}{2.828} = \frac{3}{2.828} = 1.06 \rightsquigarrow 0.8554 \Rightarrow 1 - 0.8554 = 0.1446$$

(c) What is the approximate probability that a simple random sample of 50 UCONN basketball players will reveal that at most 29% of its players are "talented?"

We have $N(p, \sqrt{\frac{p(1-p)}{n}}) = N(0.20, 0.0566)$. *Note* $np = 10$ *and* $n(1-p) = 40$.

$$z_{0.29} = \frac{0.29 - 0.20}{0.0566} = \frac{0.09}{0.0566} = 1.59 \rightsquigarrow 0.9441 \Rightarrow 1 - 0.9441 = 0.0559$$

(d) (Bonus) For what percentage values p are the approximations used in the previous parts most accurate?

The normal approximation is most accurate for p*'s which are close to* 0.50.