Name: *Caleb McWhorter — Solutions* **MAT 221**
Fall 2014 **Problem Set 8**

Formulae:

(i) The sampling distribution for a large SRS of size n ($n \geq 30$) from a population with mean μ and standard deviation σ is $N(\mu, \sigma/\sqrt{n})$. This formula always applies regardless of the sample size if the distribution is normal.

(ii) If you take an simple random sample from a large population with a proportion of p success then the number of counts $X \approx N\left(np, \sqrt{np(1-p)}\right)$ and the sample proportion of successes is $\hat{p} \approx N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$.

Problem 1: For women aged 18–24, systolic blood pressures are normally distributed with a mean of 114.8 and a standard deviation of 13.1. Hypertension is commonly defined as a systolic blood pressure above 140.

(a) Find the proportion of women who have systolic blood pressure greater than 140.

$$z_{140} = \frac{140 - 114.8}{13.1} = \frac{25.2}{13.1} = 1.92 \rightsquigarrow 0.9726 \Rightarrow 1 - 0.9726 = 0.0274$$

(b) If 5 women in that age group are randomly selected, find the probability that their mean systolic blood pressure is above 140.

$$z_{140} = \frac{140 - 114.8}{13.1/\sqrt{5}} = \frac{25.2}{5.85} = 4.31 \rightsquigarrow 1.00 \Rightarrow 1 - 1.00 = 0.0000$$

(c) If for men aged 18–24, the measurement of systolic blood pressure has a mean of 125.2 and a standard deviation of 14.2. How does this situation differ from the situation in the parts above? [Hint: What is the distribution in this case. . . read carefully!]

In this case, we do not know that the population is normally distributed. To do the calculations above, we need to know the distribution is normally distributed or that $n \geq 30$ so that the Central Limit Theorem applies.

(d) If a group of 5 men are randomly selected, can you find the probability that their mean systolic blood pressure is below 120? What about if you selected a group of 20 men? What about a group of 50 or 100 men?

We cannot do this for a group of 5 or 20 men; however, we could perform the calculation if we used a group of 50 or 100 men because then the Central Limit Theorem would apply.

Problem 2: Consider the fact that 8% of men are colorblind.

(a) Consider a SRS of 7 men. What is the probability that at least 3 of these men are colorblind?

$$P(\geq 3) = P(3) + P(4) + P(5) + P(6) + P(7) = 0.0128 + 0.0011 + 0.0001 + 0.0000 = 0.014$$

(b) Consider a SRS of 500 men. What is the approximate probability that at least 50 men in this sample are color blind?

We have $N(np, \sqrt{np(1-p)}) = N(40, 6.0663)$. *Note* $np = 40$ *and* $n(1-p) = 460$.

$$z_{50} = \frac{50 - 40}{6.0663} = \frac{10}{6.0663} = 1.65 \rightsquigarrow 0.9505 \Rightarrow 1 - 0.9505 = 0.0495$$

(c) Consider a SRS of 500 men. What is the approximate probability that less than 5% of these men are colorblind?

We have $N(p, \sqrt{\frac{p(1-p)}{n}}) = N(0.08, 0.0121)$. *Note* $np = 40$ *and* $n(1-p) = 460$.

$$z_{0.05} = \frac{0.05 - 0.08}{0.0121} = \frac{-0.03}{0.0121} = -2.48 \rightsquigarrow 0.0066$$