**Name:** *Caleb McWhorter — Solutions* **MAT 221**
**Fall 2014** **Problem Set 9**

**Problem 1:** Suppose that a student measuring the boiling temperature of a certain liquid observes the six readings and obtained the sample mean of 101.8 (in degrees Celsius). Also assume that the standard deviation for this procedure, $\sigma$, is 1.2 degrees.

(a) Construct a 95% confidence interval for the mean boiling temperature of this liquid.

*We have* $z^* = 1.960$. *Then*

$$\left(\overline{x} - z^* \frac{\sigma}{\sqrt{n}} \;,\; \overline{x} + z^* \frac{\sigma}{\sqrt{n}}\right)$$

$$\left(101.8 - 1.960\,\frac{1.2}{\sqrt{6}} \;,\; 101.8 + 1.960\,\frac{1.2}{\sqrt{6}}\right)$$

$$(100.84 \;,\; 102.76)$$

(b) Suppose you wanted to cut the margin of error to $\pm 0.5$ with a confidence interval of 95%. How many total readings would have to be included in the new sample?

$$n \geq \left(\frac{1.960 \cdot 1.2}{0.5}\right)^2 = (4.704)^2 = 22.128 \rightsquigarrow 23$$

**Problem 2:** A waitress' tips are somewhat left-skewed with mean \$4.75 and standard deviation \$2.50, A simple random sample of 40 of her tips is taken. Find the approximate probability that the sample mean of these 40 tips is greater than \$5.50. Does the skewness of the tips effect your calculations?

$$z_{5.50} = \frac{5.50 - 4.75}{2.50/\sqrt{40}} = \frac{0.75}{0.395} = 1.90 \rightsquigarrow 0.9713 \Rightarrow 1 - 0.9713 = 0.0287$$

*The skewness does not matter. We have* $n \geq 30$ *so that the Central Limit Theorem applies regardless of the distribution.*

**Problem 3:** The percentage of students at some university that wear contacts is 30%.

(a) A simple random sample of 8 university students is taken. What is the probability that exactly 5 of these students wear contact lenses?

$$0.0467$$

(b) A large sample of 150 students is taken. Find the mean $\mu$ and standard deviation $\sigma$ of the number of students in this sample who wear contact lenses.

*We have* $\mu = np = 45$ *and* $\sigma = \sqrt{np(1-p)} = 5.61$.

(c) For the larger sample in (b), use the normal approximation to estimate the probability that at least 55 in the sample wear contact lenses.

$$P(\geq 55) \approx \frac{55-45}{5.61} = \frac{10}{5.61} = 1.78 \rightsquigarrow 0.9625 \Rightarrow 1 - 0.9625 = 0.0375$$

**Problem 4:** A exam for the preparedness of students for college has mean $\mu = 75$ and standard deviation $\sigma = 8$. What is the proportion of students that scored lower than 82? If the top 3% get a scholarship, what is the minimum score a student would have to get to have a hope of getting the scholarship?

$$z_{82} = \frac{82-75}{8} = \frac{7}{8} = 0.875 \rightsquigarrow 0.8092$$

*The top 3% corresponds to the bottom 97% which corresponds to a $z$-score of* $1.88$. *But then*

$$1.88 = \frac{x-75}{8}$$

*so that* $x = 90.04$.

**Problem 5:** Many students at an "elite" university took a college exam over the course of a decade. Over this time, the exam scores had mean 558 with standard deviation 139. We suspect that the grades of these students has actually increased over time. We take a simple random sample of 100 students at an "elite" university, and find a sample mean of 585 for this elite school.

(a) State $H_0$ and $H_a$ for this situation.

$$\begin{cases} H_0 : \mu = 558 \\ \\ H_a : \mu > 558 \end{cases}$$

(b) Use a significance level of $\alpha = 0.05$ to test $H_0$ against $H_a$. Compute the test statistic and $p$-value. Determine whether $H_0$ is rejected and write your conclusion in words.

$$z_{585} = \frac{585 - 558}{139/\sqrt{100}} = \frac{27}{13.9} = 1.94 \rightsquigarrow 0.9738 \Rightarrow 1 - 0.9738 = 0.0262$$

*Therefore, we have test statistic* $1.94$ *and* $p$*-value* $0.0262$*. Because our* $p$*-value is less than the significance level of* $0.05$*, there is sufficient evidence to reject the null hypothesis.*