MAT 296, Fall 2015, Quiz 10 Solutions Score

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No calculators will be allowed on any quiz, midterm exam or on the final exam. Using or having available any calculator or other electronic device during a quiz, midterm exam or the final exam is a violation of the Academic Integrity Policy.

To receive credit you must justify your answer in full detail!

1. Determine the radius of convergence and the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{2^n\sqrt{n}}.$$

Be sure to name each "test" you use and verify the conditions needed for the test to apply.

$a_n = \dfrac{(x-5)^n}{2^n\sqrt{n}}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{2^{n+1}\sqrt{n+1}}\cdot\frac{2^n\sqrt{n}}{(x-5)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{(x-5)^n}\cdot\frac{2^n}{2^{n+1}}\cdot\frac{\sqrt{n}}{\sqrt{n+1}}\right|$$

$$= \lim_{n\to\infty}\left|x-5\cdot\frac{1}{2}\cdot\sqrt{\frac{n}{n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x-5}{2}\cdot\sqrt{\frac{n}{n+1}}\right|$$

$$= \left|\frac{x-5}{2}\right|$$

Want $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$ (using Ratio Test here)

So... $\left|\frac{x-5}{2}\right| < 1$

$-1 < \frac{x-5}{2} < 1$

$-2 < x-5 < 2$

$3 < x < 7$

The radius of convergence is...

$R = \frac{7-3}{2} = \frac{4}{2} = 2$

To find the interval of convergence: Test Endpoints

If... $x=7$: $\sum_{n=1}^{\infty}\frac{(7-5)^n}{2^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{2^n}{2^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

This diverges by the p-test.

If $x=3$: $\sum_{n=1}^{\infty}\frac{(3-5)^n}{2^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{(-2)^n}{2^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$

This series is alternating, $\frac{1}{\sqrt{n}}$ is decreasing, $\lim_{n\to\infty}\frac{(-1)^n}{\sqrt{n}} = 0$. Therefore, $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ Converges by the Alternating Series Test.

Therefore, the interval of convergence is...

$[3, 7)$