Solutions

MAT 296, Fall 2015, Quiz 4 **Score** ☐

Name: Caleb McWhorter (PRINT IN BLOCK LETTERS)
Date: Sep 25, ~~2015~~

No calculators will be allowed on any quiz, midterm exam or on the final exam. Using or having available any calculator or other electronic device during a quiz, midterm exam or the final exam is a violation of the Academic Integrity Policy.

Show all the steps in your solutions.

1. Evaluate the integral $\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx$. Simplify your answer.

Let $u = \cos x$

$du = -\sin x \, dx$

$dx = \frac{du}{-\sin x}$

Now we know $\cos^2 x + \sin^2 x = 1$ so

$\sin^2 x = 1 - \cos^2 x$

$$\int \sin^3 x \cos^2 x \, dx = \int \sin x \sin^2 x \cos^2 x \, dx$$

$$= \int \sin x (1 - \cos^2 x) \cos^2 x \, dx$$

$$= \int \sin x (1 - u^2) u^2 \frac{du}{-\sin x}$$

$$= -\int u^2 - u^4 \, du$$

$$= \int u^4 - u^2 \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3}$$

$$= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}$$

So $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \, dx =$

$$\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} \Big|_0^{\frac{\pi}{2}} =$$

$$= \left(\frac{\cos^5 \frac{\pi}{2}}{5} - \frac{\cos^3 \frac{\pi}{2}}{3}\right) - \left(\frac{\cos^5 0}{5} - \frac{\cos^3 0}{3}\right)$$

$$= 0 - \left(\frac{1}{5} - \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \boxed{2/15}$$

2. Evaluate the integral $\int x(\ln x)^2\,dx$.

u	$(\ln x)^2$	$\frac{x^2}{2}$	v
du	$\frac{2\ln x}{x}$	x	dv

$$\int x(\ln x)^2\,dx = \tfrac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx$$

Now we calculate $\int x\ln x\,dx$

u	$\ln x$	$\frac{x^2}{2}$	v
du	$\frac{1}{x}$	x	dv

$$\int x\ln x\,dx = \tfrac{1}{2}x^2\ln x - \int \frac{x}{2}\,dx$$
$$= \tfrac{1}{2}x^2\ln x - \frac{x^2}{4} + C$$

Now

$$\int x(\ln x)^2\,dx = \tfrac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx$$
$$= \tfrac{1}{2}x^2(\ln x)^2 - \left(\tfrac{1}{2}x^2\ln x - \frac{x^2}{4} + C\right)$$
$$= \tfrac{1}{2}x^2(\ln x)^2 - \tfrac{1}{2}x^2\ln x + \frac{x^2}{4} + C$$
$$= \boxed{\frac{x^2}{2}\left(\ln^2 x - \ln x + \tfrac{1}{2}\right) + C}$$