MAT 296, Fall 2015, Quiz 6 Solutions

Score

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No calculators will be allowed on any quiz, midterm exam or on the final exam. Using or having available any calculator or other electronic device during a quiz, midterm exam or the final exam is a violation of the Academic Integrity Policy.

Show all the steps in your solutions.

1. Evaluate the integral $\int \frac{2x+4}{x^3+4x}\,dx$

$$\frac{2x+4}{x^3+4x} = \frac{2x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$= \frac{A(x^2+4) + x(Bx+C)}{x(x^2+4)}$$

$$= \frac{Ax^2+4A+Bx^2+Cx}{x(x^2+4)}$$

$$= \frac{(A+B)x^2+Cx+4A}{x(x^2+4)}$$

So relating numerators...

x^2: $A+B=0$

x: $C=2$

1: $4A=4 \rightarrow A=1$
So $B=-1$

We could also use Heaviside's to get A:

$$A = \frac{2(0)+4}{0^2+4} = \frac{4}{4} = 1$$

$$\int \frac{2x+4}{x^3+4x}\,dx = \int \frac{1}{x} + \frac{-x+2}{x^2+4}\,dx$$

$$= \int \frac{1}{x} + \frac{-x}{x^2+4} + \frac{2}{x^2+4}\,dx$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

$$\int \frac{-x}{x^2+4}\,dx = \frac{-1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|u|$$

$$= -\frac{1}{2}\ln|x^2+4|$$

$u = x^2+4$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$= -1\left(\frac{1}{2}\ln|x^2+4|\right)$$

$$= -\ln\sqrt{x^2+4}$$

$$\int \frac{2}{x^2+4}\,dx = \int \frac{2}{x^2+4}\cdot\frac{1/4}{1/4}\,dx = \frac{1}{2}\int \frac{dx}{\left(\frac{x}{2}\right)^2+1}$$

$$= 2\cdot\frac{1}{2}\tan^{-1}(x/2) = \tan^{-1}(x/2)$$

So

$$\int \frac{2x+4}{x^3+4x}\,dx = \ln|x| + -\ln\sqrt{x^2+4} + \tan^{-1}(x/2) + K$$

$$= \boxed{\ln\left|\frac{x}{\sqrt{x^2+4}}\right| + \tan^{-1}(x/2) + K}$$

2. Determine whether the following improper integral is convergent or divergent, and evaluate it if it is convergent. Be sure to show all the steps.

$$\int_0^\infty \frac{x^2}{(1+x^3)^{\frac{3}{2}}}\,dx$$

$$\int \frac{x^2}{(1+x^3)^{3/2}}\,dx$$

$u = 1+x^3$

$du = 3x^2\,dx$

$dx = \dfrac{du}{3x^2}$

$$\frac{1}{3}\int \frac{1}{u^{3/2}}\,du = \frac{1}{3}\int u^{-3/2}\,du = \frac{1}{3} \quad u^{-1/2} \cdot -2 = \frac{-2}{3}u^{-1/2}$$

$$= \frac{-2}{3\sqrt{u}}$$

$$= \frac{-2}{3\sqrt{1+x^3}}$$

So

$$\lim_{b\to\infty} \int_0^b \frac{x^2}{(1+x^3)^{3/2}}\,dx = \lim_{b\to\infty} \frac{-2}{3\sqrt{1+x^3}}\Bigg|_0^b = \lim_{b\to\infty} \frac{-2}{3\sqrt{1+b^3}} - \frac{-2}{3\sqrt{1+0^3}}$$

$$= 0 - \frac{-2}{3}$$

$$= 2/3$$

Therefore, the integral converges to $2/3$.