MAT 296, Fall 2015, Quiz 7 Solutions

Score

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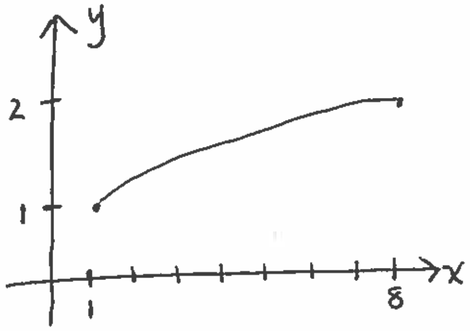
No calculators will be allowed on any quiz, midterm exam or on the final exam. Using or having available any calculator or other electronic device during a quiz, midterm exam or the final exam is a violation of the Academic Integrity Policy.

Show all the steps in your solutions.

1. Let ℓ be the segment from (1,1) to ~~(2,8)~~ (8,2) of the curve $y = x^{1/3}$. Sketch ℓ.

(a) Set up, but do not evaluate, an explicit integral for the surface area generated by revolving ℓ about the y-axis.



$$SA = 2\pi \int_1^8 x \sqrt{1 + \left(\tfrac{1}{3} x^{-2/3}\right)^2}\, dx$$

OR

$$SA = 2\pi \int_1^2 y^3 \sqrt{1 + (3y^2)^2}\, dy$$

$$y = x^{1/3} \rightleftharpoons y^3 = x$$

$$y' = \tfrac{1}{3} x^{-2/3} \qquad 3y^2 = x'$$

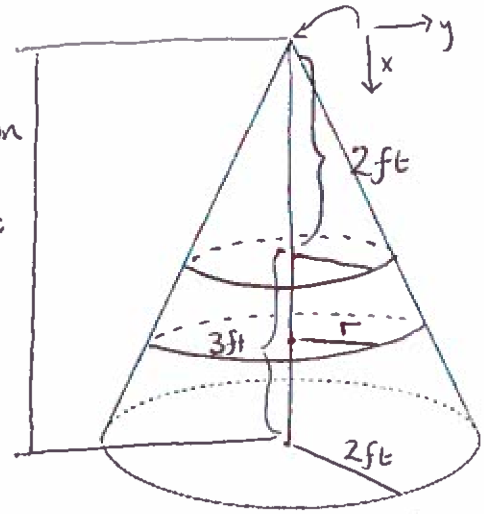
(b) Set up, but do not evaluate, an explicit integral for the surface area generated by revolving ℓ about the x-axis.

$$SA = 2\pi \int_1^8 x^{1/3} \sqrt{1 + \left(\tfrac{1}{3} x^{-2/3}\right)^2}\, dx$$

OR

$$SA = 2\pi \int_1^2 y \sqrt{1 + (3y^2)^2}\, dy$$

2. The cone shown below has radius 2ft (m) and height 5ft (m). It is filled with water to a height of 3ft (m). How much work is done pumping all the water up and out an opening at the apex of the cone? (Use ρ for the density (~~lbs/ft³~~ kg/m³) of water.)



I use S.I. units. Little changes otherwise. I use density rather than "weight" density; otherwise, remove g from solution

$W = \sum_i \text{Force} \cdot \text{distance}$ (Here $W = \int F \cdot d$)

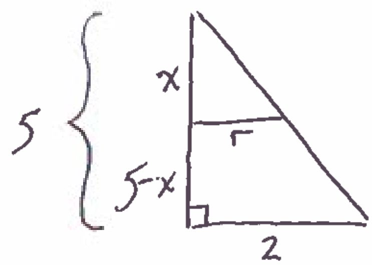
Force here is weight

$V = \pi r^2 dx = \pi \left(\frac{2}{5}x\right)^2 dx = \frac{4\pi}{25} x^2 dx$

$\text{Mass} = V\rho = \frac{4\pi}{25} \rho x^2 dx$

$\text{Weight} = g \cdot \text{mass} = g \cdot V\rho = \frac{4\pi\rho g}{25} x^2 dx$

Distance to move slice at "height" $x = x$



$\frac{x}{r} = \frac{5}{2}$

$r = \frac{2}{5}x$

g: gravity constant

ρ: density of water

$$\text{Work} = \int_2^5 \left(\frac{4\pi\rho g}{25} x^2\right) \cdot x \, dx$$

$$= \frac{4\pi\rho g}{25} \int_2^5 x^3 dx$$

$$= \frac{4\pi\rho g}{25} \left(\frac{x^4}{4}\Big|_2^5\right)$$

$$= \frac{4\pi\rho g}{25} \left(\frac{5^4}{4} - \frac{2^4}{4}\right)$$

$$= \frac{4\pi\rho g}{25} \left(\frac{625 - 16}{4}\right)$$

$$= \frac{4\pi\rho g}{25} \left(\frac{609}{4}\right)$$

$$= \boxed{\frac{609\pi\rho g}{25} \text{ J}}$$