MAT 296, Fall 2015, Quiz 8 Solutions

Score

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Date: Nov 6, 2015

No calculators will be allowed on any quiz, midterm exam or on the final exam. Using or having available any calculator or other electronic device during a quiz, midterm exam or the final exam is a violation of the Academic Integrity Policy.

Show all the steps in your solutions.

1. Determine whether the series $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$ is convergent or divergent, and justify your answer. If convergent, find the value.

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} = \sum_{n=1}^{\infty} \frac{2 \cdot 2^n}{3^n} = 2\sum_{n=1}^{\infty} \frac{2^n}{3^n} = 2\underbrace{\sum_{n=1}^{\infty} 1\cdot\left(\frac{2}{3}\right)^n}$$

Geometric with $r = 2/3$

$|r|<1$ so series converges

Convergent geometric series converge to $\frac{a}{1-r}$; $a = 1^{st}$ term, $r =$ common ratio

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} = 2\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = 2\,\frac{2/3}{1-2/3} = 2\,\frac{2/3}{1/3} = 2\cdot 2 = 4$$

$a = 2/3$

$r = 2/3$

2. Determine whether the series $\sum_{n=1}^{\infty} \frac{n+1}{6n+100}$ is convergent or divergent, and justify your answer.

$$a_n = \frac{n+1}{6n+100}$$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{6n+100} \overset{\text{L'H.}}{=} \frac{1}{6} \neq 0$ so series diverges by Divergence Test.

2. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ is convergent or divergent, and justify your answer.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} = \frac{1}{2(\ln 2)^3} + \sum_{n=3}^{\infty} \frac{1}{n(\ln n)^3}$$

So $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ Converges if and only if $\sum_{n=3}^{\infty} \underbrace{\frac{1}{n(\ln n)^3}}_{=a_n}$ Converges

1) a_n decreasing on $[3, \infty)$

2) $a_n > 0$ on $[3, \infty)$

By Integral Test, $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^3}$ Converges if and only if $\int_3^{\infty} \frac{dx}{x(\ln x)^3}$ Converges

Will ignore improperness of integral $\left.\right\}$ $\int_3^{\infty} \frac{dx}{x(\ln x)^3} = \int_{\ln 3}^{\infty} \frac{x}{x\, u^3}\, du = \underbrace{\int_{\ln 3}^{\infty} \frac{1}{u^3}\, du}_{\text{Converges by p-test}} = \left. \frac{-1}{2u^2} \right|_{\ln 3}^{\infty}$

$= \frac{1}{2(\ln 3)^2}$

$u = \ln x$

$du = \frac{dx}{x}$

$x\,du = dx$

The integral Converges, So $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^3}$ Converges So

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ Converges.