

Name: _____

Consider the function

$$f(x,y) = \begin{cases} (x^2 + y^2) \sin\left(\frac{1}{x^2 + y^2}\right) & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0) \end{cases}$$

1. (1 point) Show that $f(x,y)$ is continuous at $(0,0)$.

2. (2 points) Find f_x, f_y away from the origin.

3. (2 points) Find $f_x(0,0), f_y(0,0)$.

4. (2 points) Show that f_x, f_y are not continuous at $(0,0)$.
5. (3 points) Use the definition of differentiability to show that $f(x,y)$ is differentiable at $(0,0)$.
(Hint: since the base point is $(0,0)$, $\Delta x = x$, $\Delta y = y$)