

EXAM 2

Your Name (please PRINT): _____

Student ID Number: _____

INSTRUCTIONS

- Fill in the above items.
- There is a total of 8 problems + 1 bonus problem, for a maximum possible total value of 100 points (110 with bonus). **Make sure you have all 10 test pages (this cover page + 9 test pages).** You are responsible to check that your test booklet has all 10 pages. Alert a proctor if your copy is missing any pages.
- **Show all your work.** Only minimal credit will be given for answers without supporting work.
- **Write your answer in the box** at the bottom of pages 2-10 (except problem 1).
- **Use the back of test pages if additional space is needed,** and for scratch paper.
- **No calculators or other electronic devices; no outside notes; no outside tables** are allowed on this exam. Any use of calculators or electronic devices, or outside notes is a violation of the Academic Integrity Policy.

Do not write below this line

Question	Points	Score
1	10	
2	10	
3	15	
4	15	
5	10	
6	10	
7	15	
8	15	
9	0	
Total:	100	

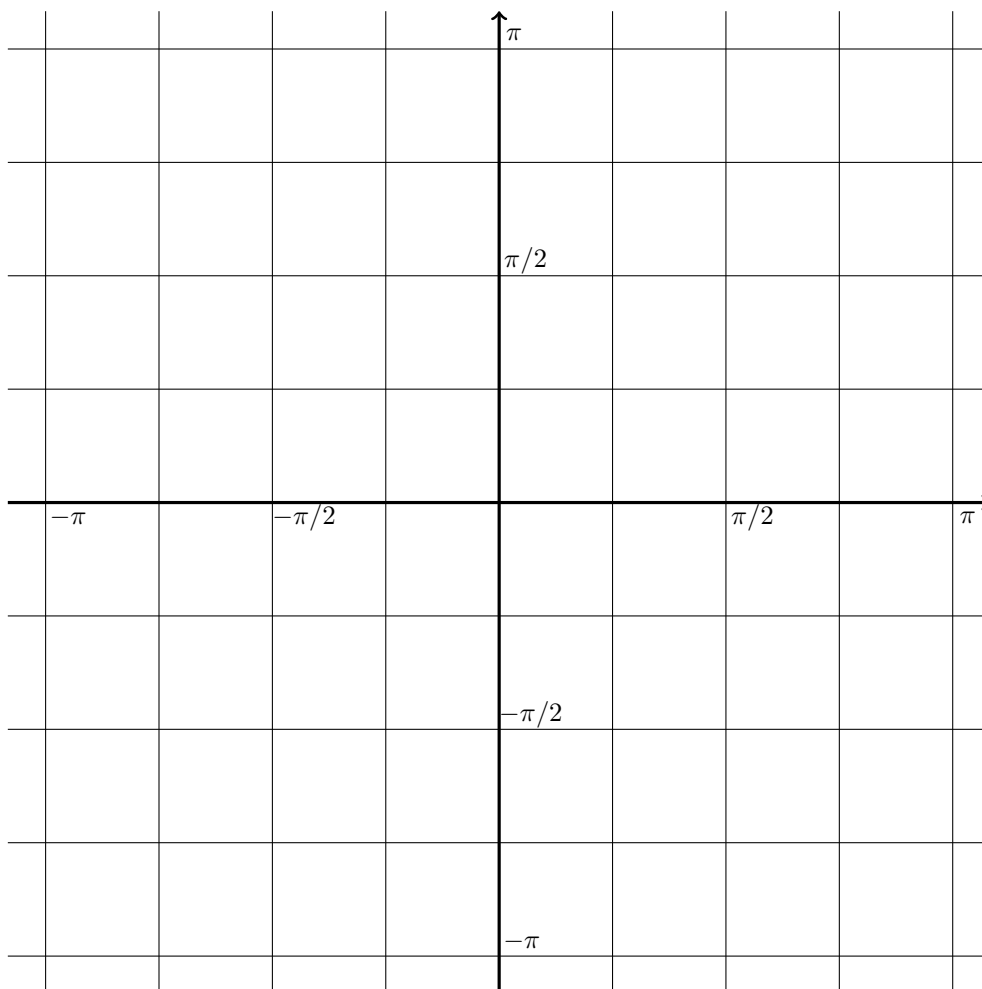
Answer for part (a):

Answer for part (b):

Answer for part (c):

1. (10 points) Sketch and label the level curves $f(x, y) = -1, 0, 1$ for the function

$$f(x, y) = \cos(x - y).$$



2. (10 points) Determine whether the following limit exists. If it does exist find the value of the limit. If it does not exist, find two paths on which the function approaches two different values.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2}$$

Answer:

3. (15 points) Find **all** of the first and second order partial derivatives of

$$f(x, y) = e^{xy} + \cos(x - y)$$

Answer:

4. Consider the surface $z = 2x^2 - xy + y^3 - x + 1$

(a) (10 points) Find the tangent plane to this surface at the point $(1, 3, 8)$.

(b) (5 points) Find the point(s) at which the tangent plane is horizontal (parallel to the xy -plane).

Answer for part (a):
Answer for part (b):

5. (10 points) A box is 4 feet wide, 3 feet long, and 5 feet high. The width is shrinking at $\frac{1}{3}$ feet per hour, the length is shrinking at $\frac{1}{5}$ feet per hour, and the height is increasing at $\frac{1}{2}$ feet per hour. At what rate is the volume of box changing?

Answer:

6. Consider $f(x, y, z) = x^2 + z \cos(y) - xyz$.

(a) (5 points) Find the direction of fastest increase of f at $(2, 0, -5)$.

(b) (5 points) Find the directional derivative of f at $(2, 0, -5)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$.

Answer for part (a):
Answer for part (b):

7. (15 points) Find the critical points of $f(x, y) = 3x^3 - 2x^2 + xy + y^2$. For each critical point determine whether it is a local maximum, local minimum or a saddle point.

Answer:

8. (15 points) Find the maximum value of $f(x, y, z) = x^2 - xy - z^2$ along the surface $4x^2 + 9y^2 + z^2 = 16$.

Answer:

9. (10 points (bonus)) Find an equation of the surface consisting of all points P for which the distance from P to the y -axis is three times the distance from P to the xy -plane. Identify the surface.

Answer: