Solutions

Quiz 2 Calculus III Fall 2015

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Solve the following problems. Explain and show work.

Q1. (3 points)

(a) Let $\vec{u} = \langle -2, 1, 2 \rangle$ and $\vec{v} = \langle 1, 0, -\frac{1}{3} \rangle$. Compute $\vec{u} + \vec{v}$.

$$\vec{u} + \vec{v} = \langle -2,1,2 \rangle + \langle 1,0,-1/3 \rangle = \langle -2+1, 1+0, 2-1/3 \rangle = \langle -1, 1, 5/3 \rangle$$

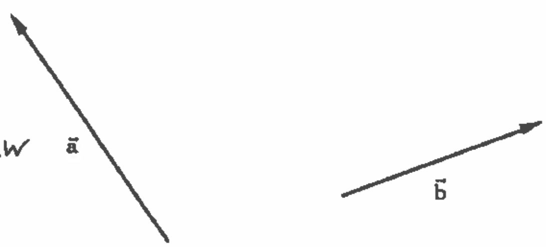
(b) Find the **unit** vector that has the same direction as $\vec{u} = \langle -2, 1, 2 \rangle$.

$$\vec{u} = \langle -2,1,2 \rangle$$

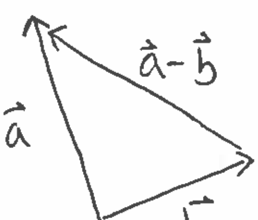
$$|\vec{u}| = \sqrt{(-2)^2 + 1^2 + 2^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

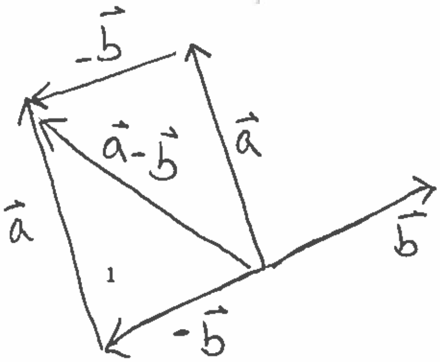
$$\frac{\vec{u}}{|\vec{u}|} = \frac{\langle -2,1,2 \rangle}{3} = \langle -2/3, 1/3, 2/3 \rangle$$

(c) Draw $\vec{a} - \vec{b}$ using the **parallelogram law**:

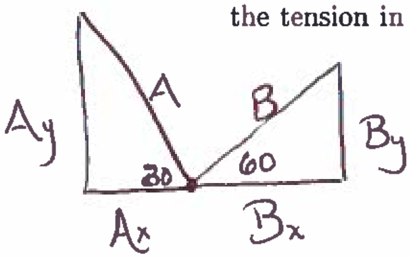


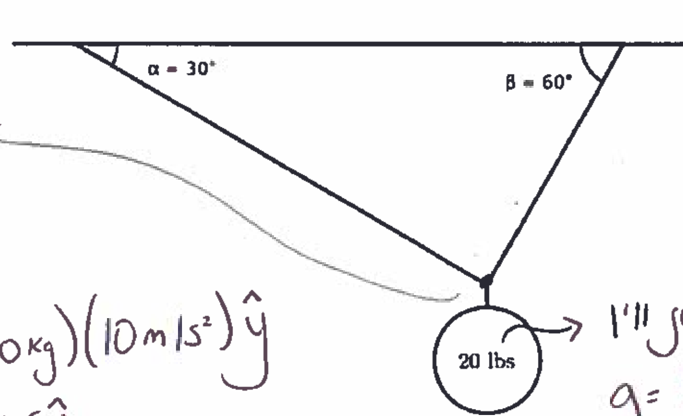
Not using the ▱-law

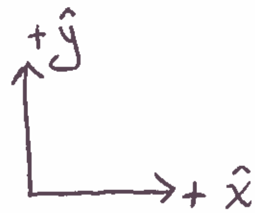




Q2. (7 points) A decoration with weight 20 lb is suspended by wires as indicated. Find the tension in each wire and the magnitude of each tension.







I'll say 20 kg

$g = 9.8\ m/s^2$

$g \approx 10\ m/s^2$

We know,

$$\vec{A} + \vec{B} = (20kg)(10\,m/s^2)\hat{y}$$

$$\vec{A} + \vec{B} = 200N\hat{y}$$

We know also...

$$\vec{A} = A_x\,\hat{x} + A_y\,\hat{y}$$

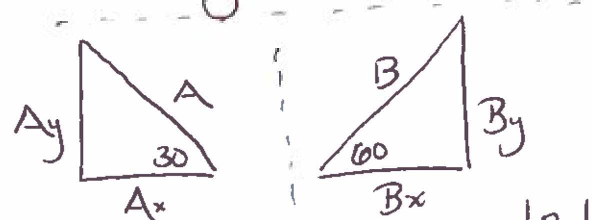
$$\vec{B} = B_x\,\hat{x} + B_y\,\hat{y}$$

Then using $\vec{A} + \vec{B} = 200N\hat{y}$ and breaking into $\hat{x}, \hat{y}$ - components

$$\vec{A} + \vec{B} = 200\hat{y}$$

$$(-|A|\cos 30°\,\hat{x} + |A|\sin 30\,\hat{y}) +$$

$$(|B|\cos 60°\,\hat{x} + |B|\sin 60°\,\hat{y}) = 200\hat{y}$$



$$\cos 30 = \frac{|A_x|}{|A|} \qquad \cos 60 = \frac{|B_x|}{|B|}$$

$$\sin 30 = \frac{|A_y|}{|A|} \qquad \sin 60 = \frac{|B_y|}{|B|}$$

$$\hat{x}: -\sqrt{3}\frac{|A|}{2} + \frac{|B|}{2} = 0$$

$$\hat{y}: \frac{|A|}{2} + \frac{\sqrt{3}\,|B|}{2} = 200$$

Multiply both equations by 2

$$\begin{cases} -\sqrt{3}|A| + |B| = 0 \\ |A| + \sqrt{3}|B| = 400 \end{cases}$$

$\times\sqrt{3}$

Subtract

$$\begin{cases} -3|A| + \sqrt{3}|B| = 0 \\ |A| + \sqrt{3}|B| = 400 \end{cases}$$

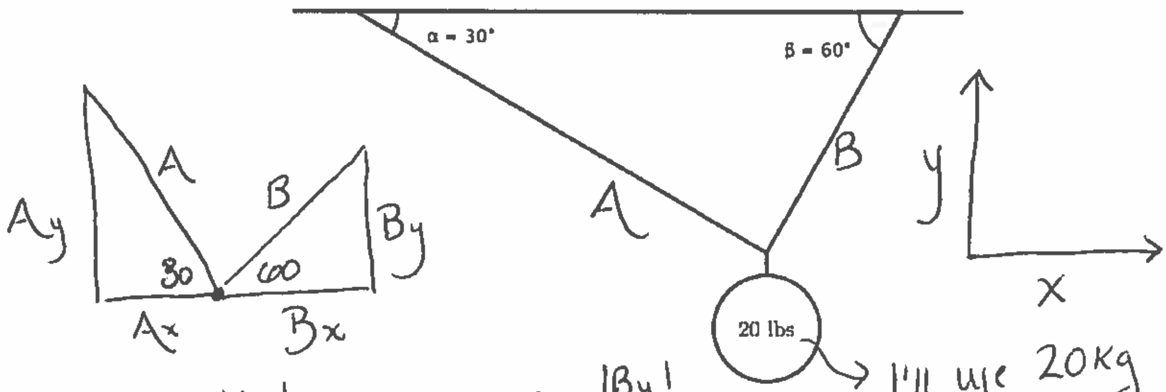
$$-4|A| = -400$$

$$\boxed{\begin{matrix} |A| = 100N \\ |B| = 300/\sqrt{3}N \end{matrix}}$$

Then

$$\boxed{\begin{matrix} \vec{A} = -50\sqrt{3}N\hat{x} + 50N\hat{y} \\ \vec{B} = \frac{150}{\sqrt{3}}N\hat{x} + 150N\hat{y} \end{matrix}}$$

Q2. (7 points) A decoration with weight 20 lb is suspended by wires as indicated. Find the tension in each wire and the magnitude of each tension.



I'll use 20Kg

$g = 9.8\ m/s^2$

$g \approx 10\ m/s^2$

$$\tan 30^\circ = \frac{|A_y|}{|A_x|} \ ; \quad \tan 60^\circ = \frac{|B_y|}{|B_x|}$$

But $|A_x| = |B_x|$

$$A_y + B_y = (10\ m/s^2)(20kg) = 200\ N$$

$$B_y = 200 - A_y$$

$$|B_x| \tan 60^\circ = 200 - |A_x| \tan 30^\circ = 200 - |B_x| \tan 30^\circ$$

$$|B_x| (\tan 60^\circ + \tan 30^\circ) = 200$$

$$|B_x| = \frac{200}{\tan 60^\circ + \tan 30^\circ} = \frac{200}{\sqrt{3} + \frac{1}{\sqrt{3}}} = 50\sqrt{3} = |A_x|$$

But $|A| = \dfrac{|A_x|}{\cos 30} = \dfrac{50\sqrt{3}}{\frac{\sqrt{3}}{2}} = \boxed{100N}$; $|B| = \dfrac{|B_x|}{\cos 60} = \dfrac{50\sqrt{3}}{1/2} = \boxed{100\sqrt{3}\ N}$

$$\vec{A} = -A_x \hat{x} + A_y \hat{y} = -|A| \cos 30^\circ \hat{x} + |A| \sin 30^\circ \hat{y} = -50\sqrt{3}N\hat{x} + 50N\hat{y}$$

$$\vec{B} = B_x \hat{x} + B_y \hat{y} = |B| \cos 60^\circ \hat{x} + |B| \sin 60^\circ \hat{y} = 50\sqrt{3}N\hat{x} + 150N\hat{y}$$