Solutions

# Quiz 3 Calculus III Fall 2015

Names: Caleb McWharter

Solve the ~~following~~ problems. Each problem is worth 5 points. Show work and explain. This is a group quiz: feel free to sit with your group mates and work together, or form an impromptu group. You will turn in one paper with all group members names on it.

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**Q1.** Consider the vectors $\vec{a} = 4\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{b} = 2\vec{j} - \vec{k}$.

**a.** Find the angle between $\vec{a}$ and $\vec{b}$.

$|\vec{a}| = \sqrt{4^2 + (-3)^2 + 1^2} = \sqrt{26}$

$|\vec{b}| = \sqrt{0^2 + 2^2 + (-1)^2} = \sqrt{5}$

$\vec{a} \cdot \vec{b} = 4(0) + (-3)2 + 1(-1) = 0 - 6 - 1 = -7$

But $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ so $\boxed{\theta = \cos^{-1}\left(\frac{-7}{\sqrt{130}}\right)}$

**b.** Find a third, non-zero vector $\vec{c}$ that is perpendicular to each of $\vec{a}$ and $\vec{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 1 \\ 0 & 2 & -1 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -3 & 1 \\ 2 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 4 & 1 \\ 0 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 4 & -3 \\ 0 & 2 \end{vmatrix}$$

$$= \boxed{1\hat{i} + 4\hat{j} + 8\hat{k}}$$

**c.** Find the area of the triangle determined by $\vec{a}$ and $\vec{b}$.

This is $\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}\sqrt{1^2 + 4^2 + 8^2}$

$= \frac{1}{2}\sqrt{81}$

$= \frac{9}{2}$

$\boxed{9/2 \text{ units}^2}$

**Q2.** (a) Find parametric equations and symmetric equations for the line that goes through $(1,-1,4)$ and is parallel to the line $x-3=\frac{1}{3}y=\frac{z-3}{4}$.

$t = x-3 = \frac{1}{3}y = \frac{z-3}{4}$

Notice this is

$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

So given line is

Parametric Equations

$$\left.\begin{array}{l} t = x-3 \\ t = \frac{1}{3}y \\ t = \frac{z-3}{4} \end{array}\right\} \rightarrow \left\{\begin{array}{l} x = t+3 \\ y = 3t \\ z = 4t+3 \end{array}\right.$$

So given line has slope $\langle 1,3,4\rangle$ (could have gotten from denominators)

So our desired line is

$\langle 1,-1,4\rangle + \langle 1,3,4\rangle t = \langle t+1, 3t-1, 4t+4\rangle$ $\left\{\begin{array}{l} x = t+1 \\ y = 3t-1 \\ z = 4t+4 \end{array}\right.$

Solving for $t$: $[t=]$ $\frac{x-1}{1} = \frac{y+1}{3} = \frac{z-4}{4}$

(b) Find the scalar equation of the plane that goes through $(1,5,1)$ and is parallel to the plane x+4y+z=3.

To be // to the given plane, must have same normal vector as given plane: $\langle 1,4,1\rangle$. But contains $\langle 1,5,1\rangle$, so

$$\langle 1,4,1\rangle \cdot \langle x-1, y-5, z-1\rangle = 0$$

$$1(x-1) + 4(y-5) + 1(z-1) = 0$$

$$x-1 + 4y-20 + z-1 = 0$$

$$x+4y+z-22 = 0$$

$$\boxed{x+4y+z=22}$$