Quiz 4 Calculus III Fall 2015

Solutions

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Solve the following problems. Each problem is worth 5 points.

Q1. Find the position vector $\mathbf{r}(t)$ of a particle with initial position $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$, initial velocity $\mathbf{v}(0) = \mathbf{k}$, and acceleration: $\mathbf{a}(t) = t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$, $t \geq 0$.

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \int \langle t, e^t, e^{-t} \rangle\,dt = \langle t^2/2, e^t, -e^{-t} \rangle + \langle c_1, c_2, c_3 \rangle$$

$$\vec{v}(0) = \langle 0,0,1 \rangle = \langle 0^2/2, e^0, -e^0 \rangle + \langle c_1, c_2, c_3 \rangle$$

$$\langle c_1, c_2, c_3 \rangle + \langle 0, 1, -1 \rangle = \langle 0, 0, 1 \rangle$$

$$\langle c_1, c_2, c_3 \rangle = \langle 0, -1, 2 \rangle$$

$$\vec{v}(t) = \langle t^2/2, e^t - 1, 2 - e^{-t} \rangle$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \int \langle t^2/2, e^t - 1, 2 - e^{-t} \rangle\,dt = \langle t^3/6, e^t - t, 2t + e^{-t} \rangle + \langle \bar{c}_1, \bar{c}_2, \bar{c}_3 \rangle$$

$$\vec{r}(0) = \langle 1, 1, 0 \rangle = \langle 0^3/6, e^0 - 0, 2(0) + e^0 \rangle + \langle \bar{c}_1, \bar{c}_2, \bar{c}_3 \rangle$$

$$\langle \bar{c}_1, \bar{c}_2, \bar{c}_3 \rangle + \langle 0, 1, 1 \rangle = \langle 1, 1, 0 \rangle$$

$$\langle \bar{c}_1, \bar{c}_2, \bar{c}_1 \rangle = \langle 1, 0, -1 \rangle$$

$$\vec{r}(t) = \langle t^3/6 + 1, e^t - t, 2t + e^{-t} - 1 \rangle$$

Q2. Find the domain and sketch the graph of the function: $f(x, y) = \sqrt{4 - 4x^2 - y^2}$.

$$4 - 4x^2 - y^2 \geq 0$$

$$4 \geq 4x^2 + y^2$$

$$1 \geq x^2 + \left(\frac{y}{2}\right)^2$$

$$z = f(x,y) = \sqrt{4 - 4x^2 - y^2}$$

$$z^2 = 4 - 4x^2 - y^2$$

$$4x^2 + y^2 + z^2 = 4$$

$$x^2 + \left(\frac{y}{2}\right)^2 + \left(\frac{z}{2}\right)^2 = 1$$