Solutions

Quiz 5 Calculus III Fall 2015

Name: Caleb McWhorter

Solve the following problems. Each problem is worth 5 points.

Q1. Either show that the limit does not exist, or prove that limit as $(x, y) \to (0, 0)$ exists and compute it.

(a) $f(x,y) = \dfrac{y^2 \sin^2 x}{x^4 + y^4}$

Along x-axis (y=0): $\lim_{(x,y)\to(0,0)} f(x,0) = 0$

Along line y=x: $\lim_{(x,y)\to(0,0)} f(x,x) = \lim_{(x,x)\to(0,0)} \dfrac{x^2 \sin^2 x}{x^4 + x^4} = \lim_{x\to 0} \dfrac{x^2 \sin^2 x}{2x^4}$

$= \lim_{x \to 0} \frac{1}{2} \dfrac{\sin^2 x}{x^2}$

$= \frac{1}{2} \lim_{x\to 0} \left(\dfrac{\sin x}{x}\right)^2$

$= 1/2$

$1/2 \neq 0$ so limit does not exist.

(b) $f(x,y) = \dfrac{y^2 \sin^2 x}{3x^2 + y^2}$

$$0 \leq \frac{y^2 \sin^2 x}{3x^2+y^2} \leq \frac{y^2 \sin^2 x}{y^2} = \sin^2 x$$

So $0 \leq \lim_{(x,y)\to(0,0)} \dfrac{y^2 \sin^2 x}{3x^2+y^2} \leq \lim_{(x,y)\to(0,0)} \sin^2 x = 0$

By Squeeze Theorem, $\lim_{(x,y)\to(0,0)} \dfrac{y^2 \sin^2 x}{3x^2+y^2} = 0$

Q2. Compute $f_x(1,2)$, $f_y(1,2)$ and $f_{xy}(1,2)$ for the function

$$f(x,y) = \frac{\sin(xy+y^2)}{x+y}$$

$$f_x = \left.\frac{y\cos(xy+y^2)(x+y) - (1)\sin(xy+y^2)}{(x+y)^2}\right|_{(1,2)} = \frac{2\cos(6)\cdot 3 - \sin 6}{9} = \frac{6\cos 6 - \sin 6}{9}$$

$$f_y = \left.\frac{(x+2y)\cos(xy+y^2)(x+y) - (1)\sin(xy+y^2)}{(x+y)^2}\right|_{(1,2)} = \frac{5\cos(6)\cdot 3 - \sin 6}{9} = \frac{15\cos 6 - \sin 6}{9}$$

$$f_{xy} = f_{yx} = \frac{\partial}{\partial y}\left[\frac{(xy+y^2)\cos(xy+y^2) - \sin(xy+y^2)}{(x+y)^2}\right]$$

$$= \frac{\left[(x+2y)\cos(xy+y^2) - (x+2y)(xy+y^2)\sin(xy+y^2) - (x+2y)\cos(xy+y^2)\right](x+y)^2 \cdots \cdots -2(x+y)\left((xy+y^2)\cos(xy+y^2) - \sin(xy+y^2)\right)}{(x+y)^4}$$

Evaluating at $(1,2)$

$$\frac{\left[\cancel{5\cos 6} - 5(6)\sin 6 - \cancel{5\cos 6}\right]9 - 2(3)\left(6\cos 6 - \sin 6\right)}{81}$$

$$= \frac{-5(6)9\sin 6 - 6(6)\cos 6 + 6\sin 6}{81}$$

$$= \frac{6\left(\overset{-5\cdot 9+1}{(\ldots)}\sin 6\right) - 36\cos 6}{81} = \frac{-264\sin 6 - 36\cos 6}{81}$$

$$= \frac{-88\sin 6 - 12\cos 6}{27}$$