Quiz 6 Calculus III Fall 2015 Solutions

Names: Caleb McWhorter

Solve the following problems. Each problem is worth 5 points. Show work and explain.

Q1.(a) Find the equation of (a) the tangent plane and (b) the normal line to the surface $xyz^2 = 6$ at $P_0(3, 2, 1)$.

$$F(x,y,z) = xyz^2 - 6$$

$$\nabla F = \langle yz^2, xz^2, 2xyz \rangle \Big|_{(3,2,1)} = \langle 2, 3, 12 \rangle$$

a.) The plane is then...

$$\langle 2,3,12 \rangle \cdot \langle x-3, y-2, z-1 \rangle = 0$$

$$2(x-3) + 3(y-2) + 12(z-1) = 0$$

$$2x + 3y + 12z = 24$$

b.)

$r(t) = \langle 2,3,12 \rangle t + \langle 3,2,1 \rangle$ or $x = 2t+3$, $y = 3t+2$, $z = 12t+1$ or $\dfrac{x-3}{2} = \dfrac{y-2}{3} = \dfrac{z-1}{12}$

Q1.(b) Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?

$$F(x,y,z) = x^2 - y^2 - z^2 - 1$$

$$\nabla F = \langle 2x, -2y, -2z \rangle$$

The plane is $x + y - z = 0$. So it has normal $\bar{n} = \langle 1, 1, -1 \rangle$

To be //, we need

$$\langle 2x, -2y, -2z \rangle = \lambda \langle 1, 1, -1 \rangle$$

for $\lambda \in \mathbb{R} \setminus \{0\}$

$\langle 2x, -2y, -2z \rangle$ is // to $\langle x, y, -z \rangle$, which is 'nicer'

So

$$\langle x, y, -z \rangle = \langle \lambda, \lambda, -\lambda \rangle$$

$x = \lambda$

$y = -\lambda$

$z = \lambda$

So given (x,y,z) is on hyperboloid...

$$(\lambda)^2 - (-\lambda)^2 - (\lambda)^2 = 1$$

$$-\lambda^2 = 1$$

$$\lambda^2 = -1$$

(over)

Impossible. So there are no such points on surface $F(x,y,z) = 0$

Q2.(a) The radius r of a circular cone is decreasing at a rate of 2 in/sec while its height h is increasing at a rate of 1.5 in/sec. At what rate is volume of the cone changing when the radius is 60in and the height is 80in?

$$V = V(r,h) = \frac{1}{3}\pi r^2 h$$

$$\frac{\partial V}{\partial r} = \frac{2}{3}\pi r h \Big|_{(60,80)} = \frac{2}{3}\pi 4800 = 3200\pi$$

$$\frac{\partial V}{\partial h} = \frac{1}{3}\pi r^2 \Big|_{(60,80)} = \frac{1}{3}\pi 3600 = 1200\pi$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$= 3200\pi(-2) + 1200\pi\left(\frac{3}{2}\right)$$

$$= -6400\pi + 1800\pi$$

$$= -4600\pi \text{ in}^3/\text{sec}$$

Q2.(b) Find all the critical points of the function: $f(x,y) = xy - 2x - 2y - x^2 - y^2$.

$f_x = y - 2 - 2x$

$f_y = x - 2 - 2y$

$$\begin{cases} y - 2 - 2x = 0 \\ x - 2 - 2y = 0 \end{cases}$$

rearranging $\begin{cases} -2x + y - 2 = 0 \\ x - 2y - 2 = 0 \end{cases}$

multiply bottom equation x2 $\begin{cases} -2x + y - 2 = 0 \\ 2x - 4y - 4 = 0 \end{cases}$

add equations: $-3y - 6 = 0$

$-3y = 6$

$y = -2$

$y - 2 - 2x = 0$

$2x = y - 2$

$2x = -2 - 2$

$2x = -4$

$x = -2$

Only critical point is $(-2,-2)$. We go further...

$f_{xx} = -2$

$f_{yy} = -2$

$f_{xy} = f_{yx} = 1$

$(-2)(-2) - 1^2 = 4 - 1 = 3 > 0$

$f_{xx} = -2 < 0$

So $(-2,-2)$ is a maximum.