Group Quiz 7 Calculus III Fall 2015 Solutions

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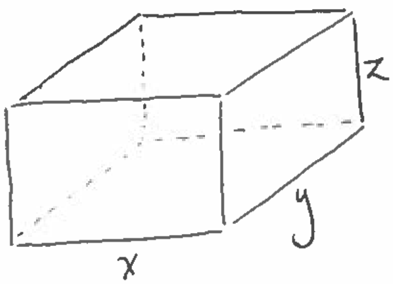
Solve the following problems. Each problem is worth 5 points.

Q1. Use the method of Lagrange Multipliers to compute the maximum volume possible for a rectangular box **with lid** that is to be made out of $12m^2$ of card board.



$$V(x,y,z) = xyz$$

$$SA(x,y,z) = 2xy + 2yz + 2xz = 2(xy + yz + xz)$$

$$\nabla V(x,y,z) = \lambda \nabla SA(x,y,z)$$

$$\langle yz, xz, xy \rangle = \lambda \langle 2(y+z), 2(x+z), 2(x+y) \rangle$$

$$\begin{cases} yz = 2\lambda(y+z) \\ xz = 2\lambda(x+z) \\ xy = 2\lambda(x+y) \end{cases}$$

$$2\lambda = \frac{yz}{y+z} = \frac{xz}{x+z} = \frac{xy}{x+y}$$

Clearly, $\lambda \neq 0$ so

$$\frac{1}{2\lambda} = \frac{y+z}{yz} = \frac{x+z}{xz} = \frac{x+y}{xy}$$

$$\frac{1}{z} + \frac{1}{y} = \frac{1}{z} + \frac{1}{x} \qquad\qquad \frac{1}{z} + \frac{1}{x} = \frac{1}{y} + \frac{1}{x}$$

$$\frac{1}{y} = \frac{1}{x} \qquad\qquad \frac{1}{z} = \frac{1}{y}$$

$$x = y \qquad\qquad z = y$$

$$2(xy + yz + xz) = 12$$

$$xy + yz + xz = 6$$

$$y^2 + y^2 + y^2 = 6$$

$$3y^2 = 6$$

$$y^2 = 2$$

$$y = \pm\sqrt{2}\ m$$

Clearly, $y > 0$ so max is

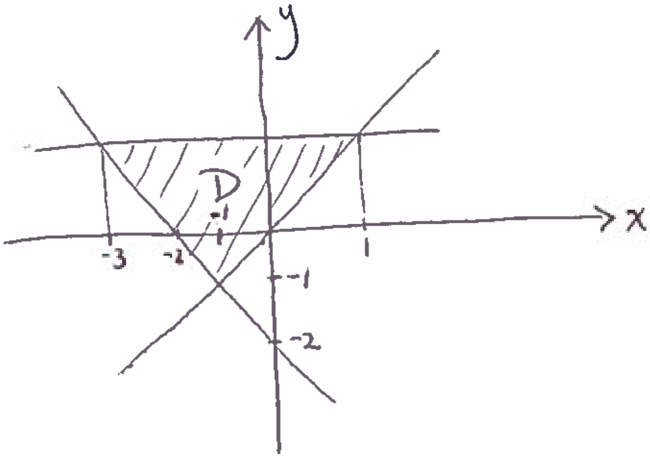
$$x = y = z = \sqrt{2}\ m$$

$$V(\sqrt{2}, \sqrt{2}, \sqrt{2}) = \boxed{2\sqrt{2}\ m^3}$$

Q2. Calculate the double integral

$$I = \iint_D y^2 \, dA$$

where $D = \{(x, y) : -1 \le y \le 1, \; -y - 2 \le x \le y\}$.



$$\iint_D y^2 dA = \int_{-1}^{1} \int_{-y-2}^{y} y^2 \, dx \, dy \quad \text{OR} = \underbrace{\int_{-3}^{-1}\int_{-x-2}^{1} y^2 dy dx}_{=2/3} + \underbrace{\int_{-1}^{1}\int_{x}^{1} y^2 dy dx}_{=2/3}$$

$$= \int_{-1}^{1} y^2 x \Big|_{-y-2}^{y} dy$$

$$= \int_{-1}^{1} y^3 - y^2(-y-2) \, dy$$

$$= \int_{-1}^{1} y^3 + y^3 + 2y^2 \, dy$$

$$= \int_{-1}^{1} 2y^3 + 2y^2 \, dy$$

$$= 2\int_{-1}^{1} y^3 + y^2 \, dy$$

$$= 2\left(\frac{y^4}{4} + \frac{y^3}{3}\right)\Big|_{-1}^{1}$$

$$= 2\left(\frac{1}{4} + \frac{1}{3} - \left(\frac{1}{4} - \frac{1}{3}\right)\right)$$

$$= 2\left(\cancel{\frac{1}{4}} + \frac{1}{3} - \cancel{\frac{1}{4}} + \frac{1}{3}\right)$$

$$= \frac{4}{3}$$