Quiz 8 Calculus III Fall 2015 (Group Quiz)

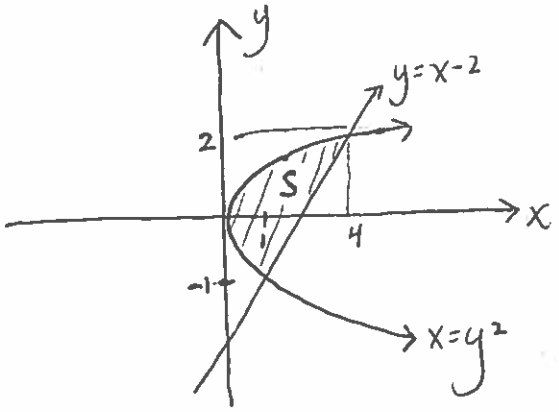
Solutions

Names: Caleb McWhorter

Solve the following problems. Each problem is worth 5 points.

Q1. Set up integral for **each** of the two orders integration. Explain which order of integration is **easiest**, then compute integral using easiest order of integration.

$$I = \iint_S y\,dA, \quad S \text{ bounded by } \quad y = x - 2, \quad x = y^2.$$



$y + 2 = x$

$y^2 = x$

$y + 2 = y^2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2$ or $y = -1$

$x = 4$ $\quad x = 1$

(1) $\int_{-1}^{2} \int_{y^2}^{y} y \, dx \, dy$

(2) $\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y \, dy \, dx + \int_1^4 \int_x^{\sqrt{x}} y \, dy \, dx$

Clearly, (1) is easier

$\int_{-1}^{2} \int_{y^2}^{y} y \, dx \, dy$

$\int_{-1}^{2} xy \Big|_{y^2}^{y} \, dy$

$\int_{-1}^{2} y^2 - y^3 \, dy$

$\left(\frac{y^3}{3} - \frac{y^4}{4} \right)_{-1}^{2}$

$\left(\frac{8}{3} - \frac{16}{4} \right) - \left(\frac{-1}{3} - \frac{1}{4} \right)$

$\frac{8}{3} - 4 + \frac{1}{3} + \frac{1}{4}$

$\frac{9}{3} - 4 + \frac{1}{4}$

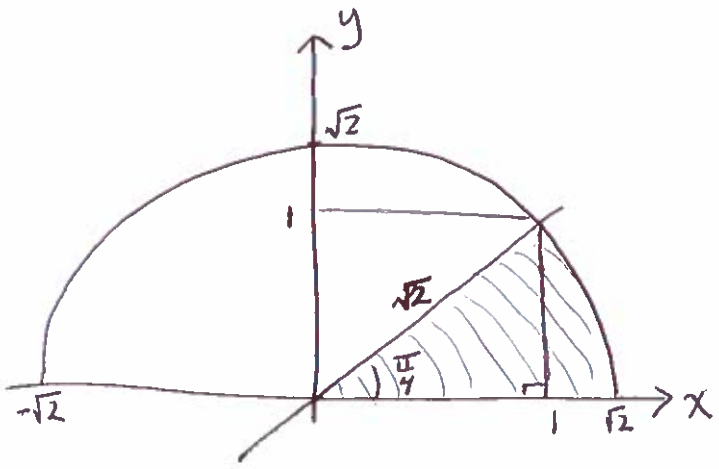
$3 - 4 + \frac{1}{4}$

$-1 + \frac{1}{4}$

$\boxed{-\frac{3}{4}}$

Q2. Compute by **changing to polar coordinates.**

$$I = \int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\,dx\,dy.$$



$x = \sqrt{2-y^2}$

$x^2 = 2 - y^2$

$x^2 + y^2 = 2$

$x = r\cos\theta$

$y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$

$$\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\,dx\,dy$$

$$\int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} (r\cos\theta + r\sin\theta)\, r\,dr\,d\theta$$

$$\int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} r^2(\cos\theta + \sin\theta)\,dr\,d\theta$$

$$\int_0^{\frac{\pi}{4}} (\cos\theta + \sin\theta) \int_0^{\sqrt{2}} r^2\,dr\,d\theta$$

$$\int_0^{\sqrt{2}} r^2\,dr \int_0^{\frac{\pi}{4}} \cos\theta + \sin\theta\,d\theta$$

$$\left(\frac{r^3}{3}\Big|_0^{2^{1/2}}\right) \cdot (\sin\theta - \cos\theta)\Big|_0^{\pi/4}$$

$$\frac{2^{3/2}}{3} \cdot \left(\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - (0-1)\right)$$

$$\frac{2\sqrt{2}}{3} \cdot 1$$

$$\boxed{\frac{2\sqrt{2}}{3}}$$