Math 397 Quiz 11

Solutions

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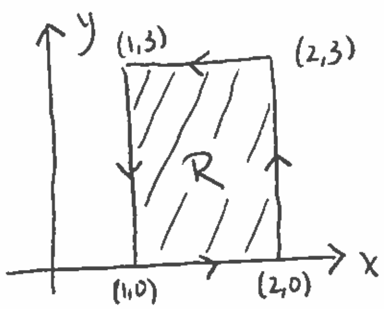
1. (10 points) Use Green's Theorem to evaluate

$$\oint_C \underbrace{(y^2 - x^2 y)}_{M}\, dx + \underbrace{(2x^2 - 2xy^2)}_{N}\, dy$$

where C is the rectangle with vertices $(1,0)$, $(2,0)$, $(2,3)$, $(1,3)$.

Assuming the above meets the condition of Green's Theorem, Green's Theorem tells us...

$$\oint_C M\,dx + N\,dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$$



$$\frac{\partial N}{\partial x} = 4x - 2y^2$$

$$\frac{\partial M}{\partial y} = 2y - x^2$$

$$\oint_C (y^2 - x^2 y)\,dx + (2x^2 - 2xy^2)\,dy =$$

$$= \iint_R (4x - 2y^2) - (2y - x^2)\, dA$$

$$= \int_1^2 \int_0^3 x^2 + 4x - 2y^2 - 2y\, dy\, dx$$

$$= \int_1^2 \left[(x^2 + 4x)y - \frac{2y^3}{3} - y^2 \right]_0^3 dx$$

$$= \int_1^2 3x^2 + 12x - 18 - 9\, dx$$

$$= \int_1^2 3x^2 + 12x - 27\, dx$$

$$= 3\int_1^2 x^2 + 4x - 9\, dx$$

$$= 3\left[\frac{x^3}{3} + 2x^2 - 9x \right]_1^2$$

$$= 3\left(\left(\frac{8}{3} + 8 - 18\right) - \left(\frac{1}{3} + 2 - 9\right) \right)$$

$$= 8 + 24 - 57 - 1 - 6 + 27$$

$$= \boxed{-2}$$