Math 397 Solutions Quiz 3

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1. (4 points) Find the arclength of $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + \frac{4}{3}t^{3/2}\mathbf{k}$ between $t = 2$ and $t = 5$.

$r'(t) = \langle 2t, 1, 2t^{1/2} \rangle$

$$L = \int_2^5 |r'(t)|\,dt = \int_2^5 \sqrt{(2t)^2 + 1^2 + (2t^{1/2})^2}\,dt$$

$$= \int_2^5 \sqrt{4t^2 + 4t + 1}\,dt$$

$$= \int_2^5 \sqrt{(2t+1)^2}\,dt$$

$$= \int_2^5 2t + 1\,dt$$

$$= (t^2 + t)\Big|_2^5 = (25 + 5) - (4 + 2) = 30 - 6 = 24$$

2. Suppose a particle moves as

$$\mathbf{r}(t) = \langle -1/t, t^2, e^t \rangle$$

(a) (3 points) Find the particle's velocity.

$r'(t) = \langle 1/t^2, 2t, e^t \rangle$

(b) (3 points) Find the particle's acceleration.

$r''(t) = \langle -2/t^3, 2, e^t \rangle$