Math 397 Solutions Quiz 4

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1. Consider the function

$$f(x,y) = \frac{3x^3}{x^2+y^2}$$

(a) (3 points) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along the line $x = 0$.

Along $x=0$, naturally $x=0$ so

$$\lim_{(0,y)\to(0,0)} f(x,y) = \lim_{(0,y)\to(0,0)} 0 = 0$$

(b) (3 points) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along the line $x = 2y$.

Along $x=2y$, naturally $x=2y$ so

$$\lim_{(2y,y)\to(0,0)} f(x,y) = \lim_{(2y,y)\to(0,0)} \frac{3(2y)^3}{(2y)^2+y^2} = \lim_{y\to 0} \frac{24y^3}{5y^2} = 0$$

(c) (4 points) Find $f_y(x,y)$

$$f_y(x,y) = \frac{\partial}{\partial y}\left(\frac{3x^3}{x^2+y^2}\right) = \frac{\partial}{\partial y}\left(3x^3(x^2+y^2)^{-1}\right)$$

$$= -3x^3(x^2+y^2)^{-2}\cdot 2y$$

$$= \frac{-6x^3y}{(x^2+y^2)^2}$$