Math 397 Solutions Quiz 5

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1. (5 points) Find the tangent plane to

$$z = y\sin x + 2y^2\cos x$$

at $(\frac{\pi}{2}, 2, 2)$.

So $\underbrace{y\sin x + 2y^2\cos x - z}_{=F(x,y,z)} = 0$

$$\nabla F = \left\langle y\cos x - 2y^2\sin x, \sin x + 4y\cos x, -1 \right\rangle \Big|_{(\frac{\pi}{2},2,2)} = \langle -8, 1, -1\rangle$$

So the tangent plane is...

$$\langle -8, 1, -1\rangle \cdot \langle x - \tfrac{\pi}{2}, y-2, z-2\rangle = 0$$

$$-8(x - \tfrac{\pi}{2}) + (y-2) + -(z-2) = 0$$

$$-8x + y - z = -4\pi$$

$$\boxed{8x - y + z = 4\pi}$$

2. (5 points) Let

$$z = \sqrt{x^2 + 2y^2} - 2xy, \qquad x = \cos t, \qquad y = \sin t$$

Find $\frac{dz}{dt}$.

Direct Substitution:

$$z = \sqrt{x^2+2y^2} - 2xy$$

$$= \sqrt{\cos^2 t + 2\sin^2 t} - 2\sin t\cos t$$

$$= \sqrt{\cos^2 t + \sin^2 t + \sin^2 t} - \sin 2t$$

$$= \sqrt{1 + \sin^2 t} - \sin 2t$$

So

$$\frac{dz}{dt} = \frac{2\sin t\cos t}{2\sqrt{1+\sin^2 t}} - 2\cos 2t$$

$$\frac{dz}{dt} = \frac{\sin 2t}{2\sqrt{1+\sin^2 t}} - 2\cos 2t$$

Chain rule:

$$\frac{dz}{dt} = \frac{\partial z}{\partial t}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{dx}{dt} = -\sin t \qquad \frac{\partial z}{\partial x} = \frac{2x}{2\sqrt{x^2+2y^2}} - 2y$$

$$\frac{dy}{dt} = \cos t \qquad \frac{\partial z}{\partial y} = \frac{4y}{2\sqrt{x^2+2y^2}} - 2x$$

Back

$$\frac{dz}{dt} = \left(\frac{2x}{2\sqrt{x^2+2y^2}} - 2y\right)\left(-\sin t\right) + \left(\frac{4y}{2\sqrt{x^2+2y^2}} - 2x\right)\left(\cos t\right)$$

$$= \left(\frac{2\cos t}{2\sqrt{\cos^2 t + 2\sin^2 t}} - 2\sin t\right)\left(-\sin t\right) + \left(\frac{4\sin t}{2\sqrt{\cos^2 t + 2\sin^2 t}} - 2\cos t\right)\left(\cos t\right)$$

$$= \left(\frac{2\cos t}{2\sqrt{1+\sin^2 t}} - 2\sin t\right)\left(-\sin t\right) + \left(\frac{4\sin t}{2\sqrt{1+\sin^2 t}} - 2\cos t\right)\left(\cos t\right)$$

$$= \frac{-2\cos t\sin t + 4\sin t\cos t}{2\sqrt{\sin^2 t + 1}} + 2\sin^2 t - 2\cos^2 t$$

$$= \frac{2\sin t\cos t}{2\sqrt{1+\sin^2 t}} + -\left(2\cos^2 t - 2\sin^2 t\right)$$

$$= \frac{\sin 2t}{2\sqrt{1+\sin^2 t}} - 2\cos 2t$$