Math 397 Solutions Quiz 6

Name: Caleb McWhorter

1. (10 points) Find the extreme values of the function

$$f(x,y) = x + 3y$$

on the region

$$\{(x,y) | x^2 + y^2 \leq 9\}.$$

$\left.\begin{array}{l} \frac{\partial f}{\partial x} = 1 \\ \frac{\partial f}{\partial y} = 3 \end{array}\right\}$ So there are no extreme values on the interior of $x^2+y^2 \leq 9$

Now we check boundary: $x^2+y^2=9$

$\nabla f(x,y) = \lambda \nabla D(x,y)$

$\langle 1, 3 \rangle = \lambda \langle 2x, 2y \rangle$

$\begin{cases} 2\lambda x = 1 \\ 2\lambda y = 3 \end{cases}$

Clearly, $x = 0 = y$ not a solution so

$2\lambda = \frac{1}{x} = \frac{3}{y}$

$x = y/3$

Know $x^2 + y^2 = 9$

$(y/3)^2 + y^2 = 9$

$y^2/9 + y^2 = 9$

$y^2 + 9y^2 = 81$

$10y^2 = 81$

$y = \pm \frac{9}{\sqrt{10}}$

So solutions are

$x_1 = \frac{3}{\sqrt{10}}$, $y_1 = \frac{9}{\sqrt{10}}$

$x_2 = \frac{-3}{\sqrt{10}}$, $y_2 = \frac{-9}{\sqrt{10}}$

$f(x_1, y_1) = \frac{3}{\sqrt{10}} + \frac{27}{\sqrt{10}} = \frac{30}{\sqrt{10}}$

$f(x_2, y_2) = \frac{-3}{\sqrt{10}} + \frac{-27}{\sqrt{10}} = \frac{-30}{\sqrt{10}}$

So max of $f(x,y)$ on $x^2+y^2 \leq 9$ is $\frac{30}{\sqrt{10}}$ and min is $\frac{-30}{\sqrt{10}}$