Math 397 Solutions Quiz 7

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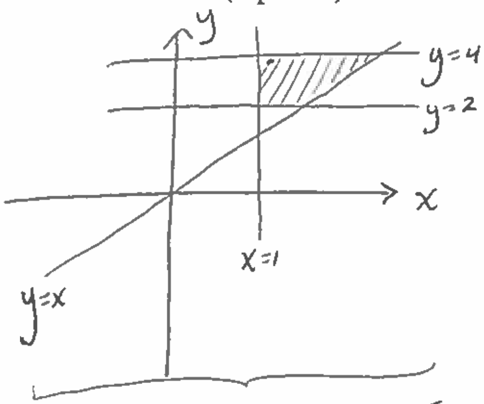
1. (5 points) Evaluate




x first y clearly easier

$$\int_2^4 \int_1^y \cos(x+y)\,dx\,dy$$

$$\int_2^4 \int_1^y \cos(x+y)\,dx\,dy$$

$$\int_2^4 \sin(x+y)\Big|_1^y \,dy$$

$$\int_2^4 \sin(2y) - \sin(1+y)\,dy$$

$$\left[-\frac{\cos 2y}{2} + \cos(1+y)\right]_2^4$$

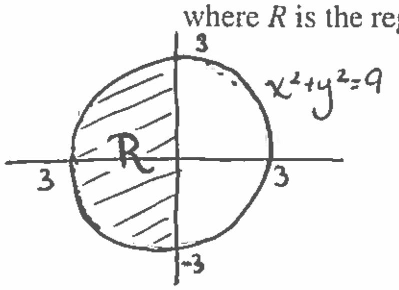
$$\left(-\frac{\cos 8}{2} + \cos 5\right) - \left(-\frac{\cos 4}{2} + \cos 3\right)$$

$$-\frac{\cos 8}{2} + \cos 5 + \frac{\cos 4}{2} - \cos 3 = \boxed{\frac{2\cos 5 - 2\cos 3 + \cos 4 - \cos 8}{2}}$$

2. (5 points) Find

$$\iint_R x - y\,dA$$

where $R$ is the region given by $x^2+y^2 \le 9$ and $x \le 0$.




In cartesian coordinates

$$\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{0} (x-y)\,dx\,dy$$

$$\int_{-3}^{0} \int_{-\sqrt{9-x^2}}^{\sqrt{9-y^2}} (x-y)\,dy\,dx$$

Clearly, easier in polar coordinates

$x = r\cos\theta$

$y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$

$$\iint_R (x-y)\,dA = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^3 (r\cos\theta - r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^3 r^2(\cos\theta - \sin\theta)\,dr\,d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\cos\theta - \sin\theta) \int_0^3 r^2\,dr\,d\theta$$

$$= \left(\int_0^3 r^2\,dr\right)\left(\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\cos\theta - \sin\theta)\,d\theta\right)$$

$$= \frac{r^3}{3}\Big|_0^3 \cdot (\sin\theta + \cos\theta)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= 9 \cdot \big((-1+0) - (1+0)\big)$$

$$= 9 \cdot -2$$

$$= \boxed{-18}$$