Math 397 Solutions Quiz 8

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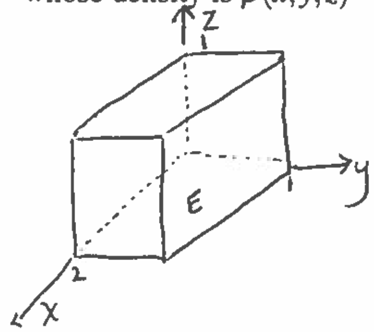
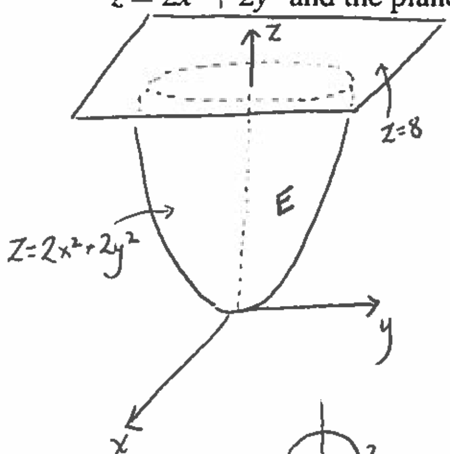
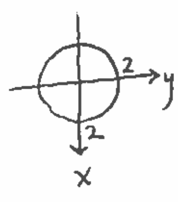
1. (5 points) Find the mass of the solid E given as $\{(x,y,z)|0 \le x \le 2,\ 0 \le y \le 1,\ 0 \le z \le 1\}$, whose density is $\rho(x,y,z) = 1 + x^2 + y - z$.



$$\text{Mass} = \iiint_E \rho(x,y,z)\, dV$$
$$= \int_0^2 \int_0^1 \int_0^1 1 + x^2 + y - z \; dz\, dy\, dx$$
$$= \int_0^2 \int_0^1 \left((1+x^2+y)z - z^2/2\right)\Big|_0^1 \, dy\, dx$$
$$= \int_0^2 \int_0^1 1 + x^2 + y - 1/2 \; dy\, dx$$
$$= \int_0^2 \int_0^1 1/2 + x^2 + y \; dy\, dx$$
$$= \int_0^2 \left((1/2 + x^2)y + y^2/2\right)\Big|_0^1 dx$$
$$= \int_0^2 1/2 + x^2 + 1/2 \; dx$$
$$= \int_0^2 1 + x^2 \, dx$$
$$= \left(x + \frac{x^3}{3}\right)\Big|_0^2 = 2 + \frac{8}{3} = \frac{6+8}{3} = \boxed{14/3}$$

2. (5 points) Find the volume of the solid E whose boundary consists of the elliptic paraboloid $z = 2x^2 + 2y^2$ and the plane $z - 8 = 0$





$$\begin{cases} z = 2x^2 + 2y^2 \\ z = 8 \end{cases}$$
$$8 = 2x^2 + 2y^2$$
$$4 = x^2 + y^2$$

Clearly, use cylindrical coordinates (all cross sections are circles)

$$V = \iiint_E dV$$
$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{2x^2+2y^2}^{8} dz\, dy\, dx$$
$$= \int_0^{2\pi} \int_0^2 \int_{2r^2}^{8} r \; dz\, dr\, d\theta$$
$$= \int_0^{2\pi} \int_0^2 r \int_{2r^2}^{8} dz\, dr\, d\theta$$
$$= \int_0^{2\pi} \int_0^2 r(8 - 2r^2)\, dr\, d\theta$$
$$= \int_0^{2\pi} d\theta \int_0^2 r(8 - 2r^2)\, dr$$
$$= 2\pi \int_0^2 8r - 2r^3\, dr$$
$$= 4\pi \int_0^2 4r - r^3\, dr$$
$$= 4\pi \left(2r^2 - \frac{r^4}{4}\right)\Big|_0^2$$
$$= 4\pi \left(8 - \frac{16}{4}\right) = 4\pi(8-4) = \boxed{16\pi}$$