Math 397 Quiz 9

Solutions

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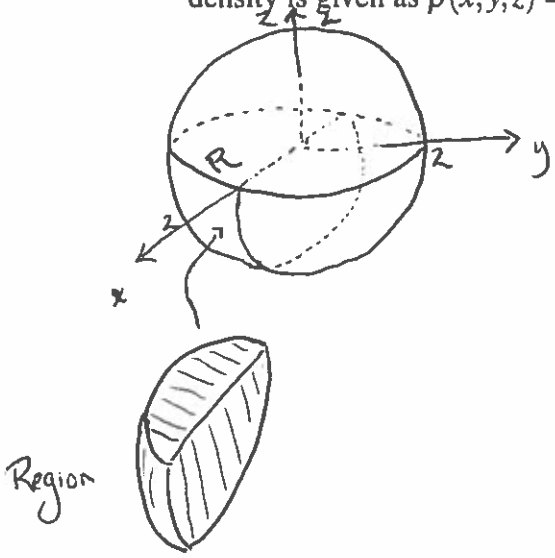
1. (10 points) Find the total mass of the region given by $x^2+y^2+z^2 \le 4$, $z \le 0$, $y \le 0$, if the density is given as $\rho(x,y,z) = x^2+y^2+z^2$.



Region

$$\iiint_R x^2+y^2+z^2 \, dV$$

$$\int_{\frac{\pi}{2}}^{\pi}\int_{\pi}^{2\pi}\int_0^2 \rho^2 \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$\int_{\frac{\pi}{2}}^{\pi}\int_{\pi}^{2\pi}\int_0^2 \rho^4 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$\int_0^2 \rho^4 \, d\rho \cdot \int_{\pi}^{2\pi} d\theta \cdot \int_{\frac{\pi}{2}}^{\pi} \sin\phi \, d\phi$$

$$\frac{\rho^5}{5}\Big|_0^2 \cdot \pi \cdot -\cos\phi\Big|_{\frac{\pi}{2}}^{\pi}$$

$$\frac{32}{5} \cdot \pi \cdot (-(-1) - -0)$$

$$\boxed{\frac{32\pi}{5}}$$