Calculus I Overview

Differentiation

- $\dfrac{d}{dx}\,(\text{constant}) = 0$
- $\dfrac{d}{dx}|x| = \dfrac{x}{|x|} = \operatorname{sgn} x,\ x \neq 0$
- $\dfrac{d}{dx}x^n = n\,x^{n-1}$
- $\dfrac{d}{dx}\sin x = \cos x$
- $\dfrac{d}{dx}\cos x = -\sin x$
- $\dfrac{d}{dx}\tan x = \sec^2 x$
- $\dfrac{d}{dx}\csc x = -\csc x \cot x$
- $\dfrac{d}{dx}\sec x = \sec x \tan x$
- $\dfrac{d}{dx}\cot x = -\csc^2 x$
- $\dfrac{d}{dx}\sin^{-1} x = \dfrac{1}{\sqrt{1-x^2}}$
- $\dfrac{d}{dx}\cos^{-1} x = \dfrac{-1}{\sqrt{1-x^2}}$
- $\dfrac{d}{dx}\tan^{-1} x = \dfrac{1}{1+x^2}$
- $\dfrac{d}{dx}\csc^{-1} x = \dfrac{-1}{|x|\sqrt{x^2-1}}$
- $\dfrac{d}{dx}\sec^{-1} x = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\dfrac{d}{dx}\cot^{-1} x = \dfrac{-1}{1+x^2}$
- $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$
- $\dfrac{d}{dx}\log_b x = \dfrac{1}{x \ln b}$
- $\dfrac{d}{dx}e^x = e^x$
- $\dfrac{d}{dx}a^x = a^x \ln a$

Product Rule: $\dfrac{d}{dx}\big(f(x)g(x)\big) = f(x)g'(x) + f'(x)g(x)$

Quotient Rule: $\dfrac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x)-f(x)g'(x)}{g(x)^2}$

Chain Rule: $\dfrac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Differentiation 'Trick'

$$
\begin{array}{ccccccccc}
\sin x & \longrightarrow & \cos x & \longrightarrow & -\sin x & \longrightarrow & -\cos x & \longrightarrow & \sin x \\
\tan x & \longleftrightarrow & \sec x & \longleftrightarrow & \sec x & & & & \\
\cot x & \longleftrightarrow & -\csc x & \longleftrightarrow & \csc x & & & &
\end{array}
$$

Integration

- $\int x^n\ dx = \frac{1}{n+1}\, x^{n+1},\ n \neq -1$
- $\int \frac{1}{x}\ dx = \ln|x|$
- $\int \sin x\ dx = -\cos x$
- $\int \cos x\ dx = \sin x$
- $\int \tan x\ dx = \ln|\sec x|$
- $\int \csc x\ dx = \ln|\csc x - \cot x|$
- $\int \sec x\ dx = \ln|\sec x + \tan x|$
- $\int \cot x\ dx = \ln|\sin x|$
- $\int \csc^2 x\ dx = -\cot x$
- $\int \sec^2 x\ dx = \tan x$
- $\int \sec x \tan x\ dx = \sec x$
- $\int \csc x \cot x\ dx = -\csc x$
- $\int e^x\ dx = e^x$
- $\int a^x\ dx = \frac{a^x}{\ln a}$
- $\int \ln x = x \ln x - x$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x$

Note that all of the above integrals are missing the integration constant, C. This has been intentionally omitted to save space.

Integral Properties:

- $\int_a^a f(x)\ dx = 0$
- $\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$
- $\int cf(x)\ dx = c\int f(x)\ dx$
- $\int f(x) \pm g(x)\ dx = \int f(x)\ dx \pm \int g(x)\ dx$
- $\int_a^b f(x)\ dx \geq 0$ if $f(x) \geq 0$ on $[a, b]$
- $m(b-a) \leq \int_a^b f(x)\ dx \leq M(b-a)$ if $m \leq f(x) \leq M$ on $[a, b]$
- $\left|\int_a^b f(x)\ dx\right| \leq \int_a^b |f(x)|\ dx$
- $\int_a^c f(x)\ dx = \int_a^b f(x)\ dx + \int_b^c f(x)\ dx$ if $a \leq b \leq c$

Identities

- $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$
- $\sin^2\theta + \cos^2\theta = 1$
- $\cos^2\theta = \frac{1+\cos 2\theta}{2}$
- $\sin^2\theta = \frac{1-\cos 2\theta}{2}$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\sin 2\theta = 2\sin\theta\cos\theta$