Problems 1-38 from Stewart's *Calculus*, page 784

D 1. $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$ Div. Test

D 2. $\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$ Limit Compare w/ $1/n$ Diverges

C 3. $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ $< \sum 1/n^2$

CC 4. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n-1}{n^2+n}$ AST $\lim = 0$ Dec. for $n \geq 3$ | Not absolute See 2

AC 5. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$ Ratio Test (get $3/8$) Conv. Ab.

AC 6. $\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$ Root Test get $3/8$

D 7. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ Integral Test $u = \ln x$

D 8. $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$ Ratio Test Get 2. Or div. Test

AC 9. $\sum_{k=1}^{\infty} k^2 e^{-k}$ Ratio Test Get $1/e$

C 10. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ Integral Test

CC 11. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n\ln(n)}$ AST

CC 12. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+25}$ AST

AC 13. $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ Ratio Test Get 0

D 14. $\sum_{n=1}^{\infty} \sin(n)$ Div. Test DNE

AC 15. $\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n+2)}$ Ratio Test Get $1/3$

D 16. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$ Limit Compare $1/n$ Get 1

D 17. $\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$ Div. Test

C 18. $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$ AST

AC 19. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$ AST

AC 20. $\sum_{k=1}^{\infty} \frac{k+5}{5^k}$ Ratio Test Get $1/5$

AC 21. $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$ Root Test Get 0

C 22. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ $< \sum \frac{1}{n^2}$ Compare

D 23. $\sum_{n=1}^{\infty} \tan(1/n)$ Limit Compare $1/n$ L'Hô $\sec^2(1/x)(-1/x^2)$ Tan' $\sec^2(1/x)$ $= 1^2$

C 24. $\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2+4n}$ Compare $1/n^2$

AC 25. $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$ Ratio. Get 0

AC 26. $\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$ Ratio Test Get $1/5$

C 27. $\sum_{k=1}^{\infty} \frac{k\ln(k)}{(k+1)^3}$ $< \frac{k \ln k}{k^3} = \left(\frac{\ln k}{k^2}\right)$ Int Test

C 28. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ Integral Test Compare to e/n^2

C 29. $\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n\sqrt{n}}$ $< \frac{\pi/2}{n^{3/2}}$ $\sum 1 \frac{\pi/2}{n^{3/2}}$ Comparison $= \frac{\pi}{2} \sum 1/n^{3/2}$

C 30. $\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$ AST $\lim = 0$ Dec. for $j > 5$

D 31. $\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$ Div. Test

AC 32. $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ Root Test. Get 0

C 33. $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$ Limit Compare $1/n\sqrt{n}$ Get 1

D 34. $\sum_{n=1}^{\infty} \frac{1}{n + n\cos^2(n)}$ $\geq$ Compare $\frac{1}{n+n} = \frac{1}{2n}$

AC 35. $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ Root Test $= \left(\frac{1}{(n+1)/n}\right)^n = \frac{1}{(1+1/n)^n} = 1/e < 1$

C 36. $\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$ Compare $(e^{\ln\ln n})^{\ln n} = (e^{\ln n})^{\ln\ln n} = n^{\ln\ln n}$ > 2 when n large

AC 37. $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$ $(\ln n)^{\ln n} > n^2$ Root Test $1 - 1 = 0$ So Series $< 1/n^2$

D 38. $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)$ Limit Compare OR $b_n = 1/n$. Get $1 \cdot \ln 2$

$$= \frac{1}{2^{(n-1)/n} + 2^{(n-2)/n} + 2^{(n-3)/n} + \cdots + 2^{1/n} + 1} \geq \frac{1}{2n}$$