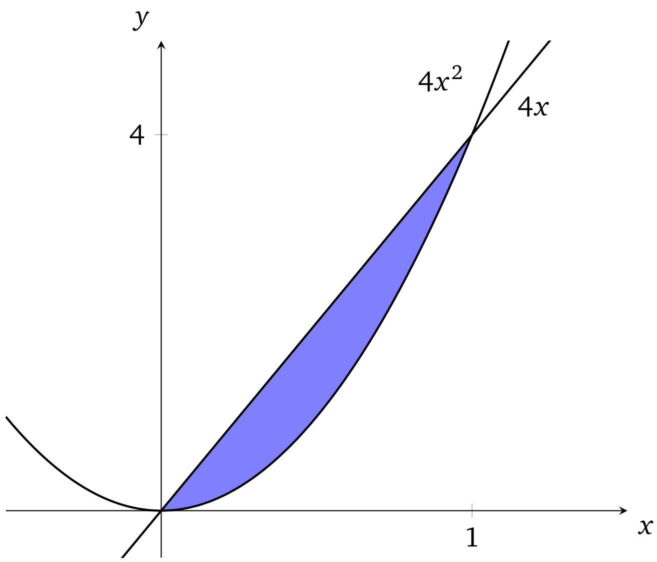
Name: *Caleb McWhorter — Solutions* **Quiz 1: Areas & Volumes**
MAT 296 **Spring 2015**

(i) Sketch the region bound by the curves $y = 4x$ and $y = 4x^2$.



$$y = 4x \quad \Leftrightarrow \quad x = \frac{y}{4}$$

$$y = 4x^2 \quad \Leftrightarrow \quad x = \frac{\sqrt{y}}{2}$$

(ii) Find the *area* of this region.

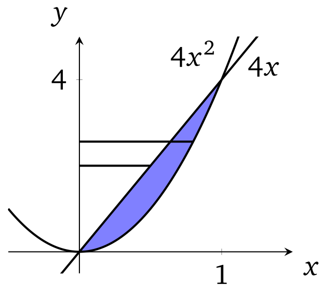
$$\int_0^1 (4x - 4x^2) = 4\int_0^1 (x - x^2)\,dx = 4\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 4\left[\left(\frac{1}{2} - \frac{1}{3}\right) - 0\right] = 4\left(\frac{3}{6} - \frac{2}{6}\right) = 4 \cdot \frac{1}{6} = \frac{2}{3}$$

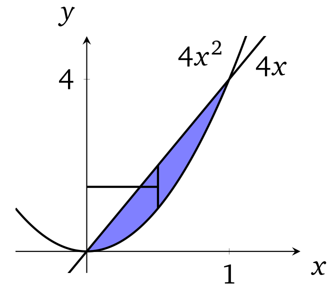
OR

$$\int_0^4 \left(\frac{\sqrt{y}}{4} - \frac{y}{4}\right) dy = \frac{y^{3/2}}{3} - \frac{y^2}{8}\bigg|_0^4 = \left(\frac{4^{3/2}}{3} - \frac{4^2}{8}\right) - 0 = \frac{8}{3} - 2 = \frac{8}{3} - \frac{6}{3} = \frac{2}{3}$$

(iii) Find the volume obtained by rotating this region about the y-axis.

Note that $y = 4x \Leftrightarrow x = y/4$ *and* $y = 4x^2 \Leftrightarrow x = \frac{\sqrt{y}}{2}$.



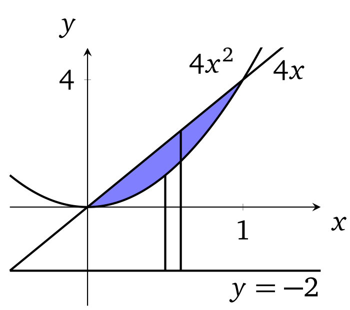


$$\pi \int_0^4 \left(\frac{\sqrt{y}}{2}\right)^2 - \left(\frac{y}{4}\right)^2 \, dy$$
$$\pi \int_0^4 \frac{y}{4} - \frac{y^2}{16} \, dy$$
$$\pi \left[\frac{y^2}{8} - \frac{y^3}{48}\right]_0^4$$
$$\pi \left[2 - \frac{4}{3}\right]$$
$$\pi \left[\frac{6}{3} - \frac{4}{3}\right] = \frac{2\pi}{3}$$

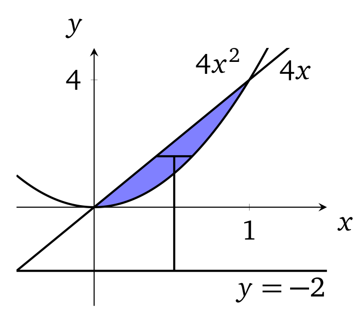
OR

$$2\pi \int_0^1 x(4x - 4x^2) \, dx$$
$$4 \cdot 2\pi \int_0^1 (x^2 - x^3) \, dx$$
$$8\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$
$$8\pi \left[\left(\frac{1}{3} - \frac{1}{4}\right) - 0\right]$$
$$8\pi \left[\frac{4}{12} - \frac{3}{12}\right] = \frac{2\pi}{3}$$

(iv) Set up the an integral for finding the volume obtained by rotating this region about the line $y = -2$. **You only need to set this integral up. You do not need to find the value.**



OR



$$\pi \int_0^1 (4x + 2)^2 - (4x^2 + 2)^2 \, dx$$

$$2\pi \int_0^4 (y + 2)\left(\frac{\sqrt{y}}{2} - \frac{y}{4}\right) dy$$