Name: *Caleb McWhorter — Solutions* **Quiz 2: Integration by Parts**

MAT 296 **Spring 2015**

Problem 1: Find

$$\int \frac{\ln x}{x^2}\,dx$$

$\ln x$	$-\frac{1}{x}$
$\frac{1}{x}$	$\frac{1}{x^2}$

$$\begin{aligned}\int \frac{\ln x}{x^2}\,dx &= -\frac{1}{x}\ln x - \int \frac{-1}{x^2}\,dx\\ &= -\frac{\ln x}{x} + \int \frac{dx}{x^2}\\ &= -\frac{\ln x}{x} - \frac{1}{x} + C\\ &= -\frac{1+\ln x}{x} + C\end{aligned}$$

Problem 2: Find

$$\int (x+2)\cos x\,dx$$

$x+2$	$\sin x$
1	$\cos x$

$$\begin{aligned}\int (x+2)\cos x\,dx &= (x+2)\sin x - \int \sin x \cdot 1\,dx\\ &= (x+2)\sin x - \int \sin x\,dx\\ &= (x+2)\sin x - (-\cos x) + C\\ &= (x+2)\sin x + \cos x + C\end{aligned}$$

Problem 3: Evaluate

$$\int_1^4 \ln\sqrt{x}\,dx$$

$\ln\sqrt{x} = \frac{1}{2}\ln x$	x
$\frac{1}{2x}$	1

$$\begin{aligned}\int \ln\sqrt{x}\,dx &= x\ln\sqrt{x}\Big|_1^4 - \int_1^4 x\cdot\frac{1}{2x}\,dx\\ &= x\ln\sqrt{x}\Big|_1^4 - \int_1^4 \frac{1}{2}\,dx\\ &= \left(x\ln\sqrt{x} - \frac{x}{2}\right)\Big|_1^4\\ &= (4\ln\sqrt{4} - \ln\sqrt{1}) - (\frac{1}{2}\cdot 4 - \frac{1}{2}\cdot 1)\\ &= 4\ln 2 - 2 + \frac{1}{2}\\ &= 4\ln 2 - \frac{3}{2}\end{aligned}$$

Extra Credit: Do NOT attempt this unless you have finished the rest of the quiz and are confident with your answers!

Find

$$\int \sqrt{e^{6x}+e^{4x}}\,dx$$

$$\sqrt{e^{6x}+e^{4x}} = \sqrt{e^{4x}(e^{2x}+1)} = e^{2x}\sqrt{e^{2x}+1}$$

Then

$$\int \sqrt{e^{6x}+e^{4x}}\,dx = \int e^{2x}\sqrt{e^{2x}+1}\,dx$$

Let $u = e^{2x}$ *so that* $du = 2e^{2x}\,dx \Leftrightarrow dx = \frac{du}{2e^{2x}} = \frac{du}{2u}$. *But then*

$$\int e^{2x}\sqrt{e^{2x}+1}\,dx = \int u\sqrt{u+1}\,\frac{du}{2u}$$

$$= \frac{1}{2}\int \sqrt{u+1}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{3}(u+1)^{3/2}+C$$

$$= \frac{1}{3}(u+1)^{3/2}+C$$

$$= \frac{1}{3}(e^{2x}+1)^{3/2}+C$$