Name: *Caleb McWhorter — Solutions* **Quiz 3: Partial Fractions**

MAT 296 **Spring 2015**

For each of the following problems, indicate the correct partial fraction decomposition:

Problem 1:

$$\frac{x-7}{x^2(x+3)}$$

(i) $\frac{A}{x}+\frac{Bx+C}{x^2}+\frac{D}{x+3}$

(ii) $\frac{A}{x^2}+\frac{B}{x+3}$

(iii) $\frac{Ax+B}{x^2}+\frac{C}{x+3}$

(iv) $\frac{A}{x}+\frac{B}{x}+\frac{C}{x+3}$

(v) None of the above ✓

Problem 2:

$$\frac{x^2+x+17}{x^2(3x+5)^2}$$

(i) $\frac{A}{x^2}+\frac{B}{(3x+5)^2}$

(ii) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x+5}+\frac{D}{(3x+5)^2}$ ✓

(iii) $\frac{A}{x}+\frac{Bx+C}{x^2}+\frac{D}{3x+5}+\frac{Ex+F}{(3x+5)^2}$

(iv) $\frac{A}{x}+\frac{Bx+C}{x^2}+\frac{Dx+E}{3x+5}+\frac{Fx+G}{(3x+5)^2}$

(v) None of the above

Problem 3:

$$\frac{2x+13}{x^2(2x^2+5)^2}$$

(i) $\frac{A}{x^2}+\frac{B}{2x^2+5}+\frac{C}{(2x^2+5)^2}$

(ii) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x^2+5}+\frac{D}{(2x^2+5)^2}$

(iii) $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{2x^2+5}+\frac{Ex+F}{(2x^2+5)^2}$ ✓

(iv) $\frac{A}{x}+\frac{Bx+C}{x^2}+\frac{Dx+E}{2x^2+5}+\frac{Fx+G}{(2x^2+5)^2}$

(v) None of the above

Problem 4: A student is trying to solve $\int \frac{x^6-2x^5+6x+7}{x^2(x-5)}\,dx$. As their first step, they break the fraction into

$$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-5}$$

Is this a correct first step? If not, what should they have done first?

No, the degree of the numerator is greater than the or equal to degree of the denominator. The student needs to polynomial long divide first.

Problem 5: Integrate the following:

$$\int \frac{x+2}{x^2(x+1)}\,dx$$

$$\frac{x+2}{x^2(x+1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$$

$$= \frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$$

$$= \frac{Ax^2+Ax+Bx+B+Cx^2}{x^2(x+1)}$$

$$= \frac{(A+C)x^2+(A+B)x+B}{x^2(x+1)}$$

This gives a system of equations:

$$\begin{aligned} A+C &= 0 \\ A+B &= 1 \\ B &= 2 \end{aligned}$$

But $B=2$ *and as* $A+B=1$, *we have* $A=-1$. *Now* $A+C=0$ *so that* $C=1$. *Therefore,*

$$\int \frac{x+2}{x^2(x+1)}\,dx = \int \frac{-1}{x}+\frac{2}{x^2}+\frac{1}{x+1}\,dx = -\ln|x|-\frac{2}{x}+\ln|x+1|+K$$