Name: *Caleb McWhorter — Solutions* **Quiz 4: Partial Fractions & Improper Integrals**
MAT 296 **Spring 2015**

Problem 1: Integrate the following:

$$\int \frac{3x+1}{(x-2)(x+5)}\,dx$$

$$\begin{aligned}\frac{3x+1}{(x-2)(x+5)} &= \frac{A}{x-2}+\frac{B}{x+5}\\ &= \frac{A(x+5)+B(x-2)}{(x-2)(x+5)}\\ &= \frac{Ax+5A+Bx-2B}{(x-2)(x+5)}\\ &= \frac{(A+B)x+(5A-2B)}{(x-2)(x+5)}\end{aligned}$$

This gives a system of equations

$$\begin{aligned}A+B&=3\\ 5A-2B&=1\end{aligned}$$

Multiplying the first equation by 2 gives $2A+2B=6$*, and then adding this to the second equation gives* $7A=7$ *so that* $A=1$*. But as* $A+B=3$*, we have* $B=2$*. We could also use Heaviside's Method:*

$$A=\frac{3(2)+1}{2+5}=1$$

$$B=\frac{3(-5)+1}{-5-2}=2$$

Then we have

$$\int \frac{3x+1}{(x-2)(x+5)}\,dx=\int\left(\frac{1}{x-2}+\frac{2}{x+5}\right)dx=\ln|x-2|+2\ln|x+5|+K$$

Problem 2: Find the value of the following integral or show that the integral does not converge.

$$\int_2^\infty \frac{dx}{\sqrt[7]{(x-1)^2}}\,dx$$

$$\lim_{b\to\infty}\int_2^b \frac{dx}{\sqrt[7]{(x-1)^2}} = \lim_{b\to\infty}\int_5^b (x-3)^{-2/7}\,dx = \lim_{b\to\infty}\frac{7}{5}(x-1)^{5/7}\Big|_2^b = \lim_{b\to\infty}\frac{7}{5}(b-1)^{5/7} - \frac{7}{5}(2-1)^{5/7} = \infty$$

Therefore, $\int_2^\infty \frac{1}{\sqrt[4]{(x-1)^2}}\,dx$ *diverges.*

Problem 3: Use the Comparison Test to determine if the following integral is convergent or divergent.

$$\int_1^\infty \frac{6x}{3x^5+1}\,dx$$

$$\int_1^\infty \frac{6x}{3x^5+1}\,dx \le \int_1^\infty \frac{6x}{3x^5}\,dx = 2\int_1^\infty \frac{dx}{x^4}$$

The integral $\int_1^\infty \frac{dx}{x^4}$ *converges by the p-test. Therefore,* $\int_1^\infty \frac{6x}{3x^5+1}\,dx$ *converges by the Comparison Test.*