Name: *Caleb McWhorter — Solutions* **Quiz 5:** **Improper Integrals, Arc Length, Surface Area**
MAT 296 **Spring 2015**

Problem 1: Set up the integral for ***but do not evaluate*** the following:

(i) The length of the curve $x = \frac{2}{5}\sqrt{(y-1)^5}$ between $1 \leq y \leq 3$.

Note $x = \frac{2}{5}(y-1)^{5/2}$ *and* $x' = (y-1)^{3/2}$. *We have* $\sqrt{1+(x')^2} = \sqrt{1+((y-1)^{3/2})^2} = \sqrt{1+(y-1)^3}$. *Then*

$$L = \int_1^3 \sqrt{1+(y-1)^3}\, dy$$

(ii) The surface area generated by $f(x) = 3 + \sin x$ between $0 \leq x \leq \pi$ rotated about the x-axis.

We have $f(x) = 2 + \sin x$ *and* $f'(x) = \cos x$. *Then* $\sqrt{1+(f')^2} = \sqrt{1+\cos^2 x}$. *Then*

$$SA = 2\pi \int_0^\pi (2+\sin x)\sqrt{1+\cos^2 x}\, dx$$

Problem 2: Determine if the following integral is convergent or divergent. If it converges, find the value.

$$\int_1^\infty \frac{1+\sin^2(x^3)}{\sqrt{x}}\, dx$$

We know that $0 \leq |\sin^2(x^3)| \leq 1$. *But then*

$$\int_1^\infty \frac{dx}{\sqrt{x}} = \int_1^\infty \frac{1+0}{\sqrt{x}}\, dx \leq \int_1^\infty \frac{1+\sin^2(x^3)}{\sqrt{x}}\, dx$$

But $\int_1^\infty \frac{dx}{\sqrt{x}}$ *diverges by the p-test. Therefore,* $\int_1^\infty \frac{1+\sin^2(x^3)}{\sqrt{x}}\, dx$ *diverges by the Comparison Test.*

Problem 3: Show whether this integral converges or diverges. If it converges, find the value.

$$\int_1^\infty \frac{dx}{2x^2+x}$$

[Hint: Try using partial fractions. Be ***very*** careful evaluating this integral!]

$$\lim_{b\to\infty}\int_1^b \frac{dx}{2x^2+x} = \lim_{b\to\infty}\int_1^b \frac{dx}{x(2x+1)}$$

Now we have partial fraction decomposition

$$\begin{aligned}\frac{1}{x(2x+1)} &= \frac{A}{x}+\frac{B}{2x+1}\\ &= \frac{A(2x+1)+Bx}{x(2x+1)}\end{aligned}$$

Then we have $A(2x+1)+Bx = 2Ax + A + Bx = (2A+B)x + A$ *so that we have system of equations*

$$\begin{aligned}2A+B&=0\\ A&=1\end{aligned}$$

implying that $B=-2$. *Therefore,*

$$\begin{aligned}\lim_{b\to\infty}\int_1^b \frac{dx}{2x^2+x} &= \lim_{b\to\infty}\int_1^b \frac{1}{x}+\frac{-2}{2x+1}\,dx\\ &= \lim_{b\to\infty}\left(\ln|x| - \frac{2\ln|2x+1|}{2}\right)\Bigg|_1^b\\ &= \lim_{b\to\infty}\left(\ln|x| - \ln|2x+1|\right)\Big|_1^b\\ &= \lim_{b\to\infty}\ln\left(\frac{x}{2x+1}\right)\Bigg|_1^b\\ &= \lim_{b\to\infty}\ln\left(\frac{b}{2b+1}\right) - \ln\left(\frac{1}{2(1)+1}\right)\\ &= \ln(1/2)-\ln(1/3)\\ &= -\ln 2+\ln 3\\ &= \ln\left(\frac{3}{2}\right)\end{aligned}$$

Note: We could also have used Heaviside's Method:

$$\begin{aligned}A &= \frac{1}{2(0)+1} = 1\\ B &= \frac{1}{-1/2} = -2\end{aligned}$$