Name: *Caleb McWhorter — Solutions* **Quiz 6: Sequences & Series**
MAT 296 **Spring 2015**

Problem 1:

(i) Find the first 5 terms of the sequence $a_n = \sin(1/n)$.

$$a_1 = \sin(1), \quad a_2 = \sin(1/2), \quad a_3 = \sin(1/3), \quad a_4 = \sin(1/4), \quad a_5 = \sin(1/5)$$

(ii) Give a formula for the following sequence: $3, 7, 15, 31, 63, \ldots$

$a_n = 2^{n+2} - 1$ *or* $a_0 = 3$ *with* $a_{n+1} = 2a_n + 1$.

Problem 2: Find the limit of the following sequences. If the sequence does not converge, write DNE.

(i) $a_n = \dfrac{7n^4 - 2n^2 - n}{3n^4 - n^3 + 9n^2}$ — $7/3$

(ii) $a_n = \dfrac{\ln(12n)}{5n}$ — 0

(iii) $a_n = \cos(\pi n) - \dfrac{1}{n^4}$ — DNE

(iv) $a_n = \ln(6n^4) - 4\ln n$ — $ln(6)$

Problem 3: Mark the following statements True or False.

(i) The series $\displaystyle\sum_{n=1}^{\infty} \frac{3n+6}{2n^2}$ converges. — *False*

(ii) The series $\cos 1 + \cos(1/2) + \cos(1/4) + \cos(1/16) + \cdots$ diverges. — *True*

(iii) If a_n is a sequence and $\lim\limits_{n\to\infty} a_n = 0$, then $\sum a_n$ converges. — *False*

(iv) The series $5 - 5 + 5 - 5 + 5 - 5 + \cdots$ converges to 0. — *False*

Bonus: Recall that given the sequence $s_n = \left(1 + \dfrac{1}{n}\right)^n$ that we have $\lim\limits_{n\to\infty} s_n = e$. Find the limit of the sequence $a_n = \left(\dfrac{2n+4}{2n}\right)^{4n}$.

$$\lim_{n\to\infty}\left(\frac{2n+6}{2n}\right)^{3n} = \lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{3n} = \lim_{n\to\infty}\left(1+\frac{1}{n/3}\right)^{3n} = \lim_{n\to\infty}\left[\left(1+\frac{1}{n/3}\right)^{n/3}\right]^9 = \left[\lim_{n\to\infty}\left(1+\frac{1}{n/3}\right)^{n/3}\right]^9 = e^9$$