Name: *Caleb McWhorter — Solutions* **Quiz 7: Series Test & Geometric Series**
MAT 296 **Spring 2015**

Problem 1: Consider the following series. If the series converges, explain why and find the sum. If the series diverges, explain why.

$$\sum_{k=2}^{\infty} \frac{1}{3}\left(\frac{2}{e^2}\right)^k$$

The series is geometric. We have $r = 2/e^2$. Because $e > 2$, $|r| < 1$, so the series converges. We have

$$\sum_{k=2}^{\infty} \frac{1}{3}\left(\frac{2}{e^2}\right)^k = \frac{\frac{1}{3}\left(\frac{2}{e^2}\right)^2}{1-\frac{2}{e^2}} = \frac{4}{3e^2(e^2-2)}$$

Problem 2: Use the Comparison Test to prove the convergence or divergence of the following series:

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^4+2}$$

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^4+2} < \sum_{n=1}^{\infty} \frac{1}{n^4+2} < \sum_{n=1}^{\infty} \frac{1}{n^4}$$

The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$ *converges by the p-test. Therefore,* $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^4+2}$ *converges by the Comparison Test.*

Problem 3: Determine whether the following series diverges or converges. [Hint: Divergence Test.]

$$\sum_{n=3}^{\infty} \frac{n^4+n^2-7}{3n^4-n^3+6}$$

$$\lim_{n\to\infty} \frac{n^4+n^2-7}{3n^4-n^3+6} = \frac{1}{3} \neq 0$$

Therefore, the series diverges by the Divergence Test.