Name: *Caleb McWhorter — Solutions* **Quiz 9: Polar Integration**

MAT 296 **Spring 2015**

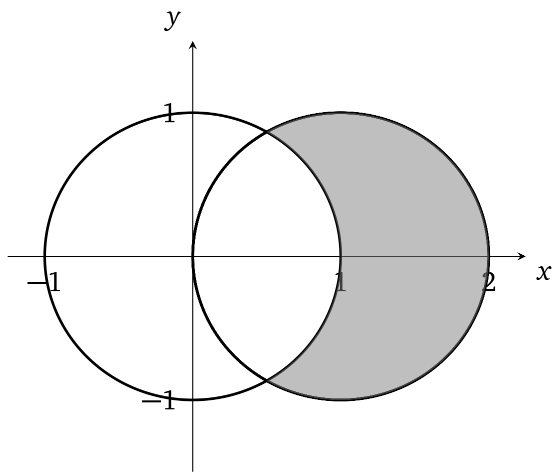
Formulae: Recall the formula you ***certainly*** already have memorized for the final:

$$A = \frac{1}{2}\int_a^b r(\theta)^2\,d\theta$$

and this formula

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

Problem 1: Find the area of the region inside the circle $r = 2\cos\theta$ but outside the circle $r = 1$. [You should draw the picture of these curves first.]



$$2\cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \simeq -\frac{\pi}{3}$$

$$\frac{1}{2}\int_{-\pi/3}^{\pi/3} 1^2\,d\theta = \frac{1}{2}\,\theta\Big|_{-\pi/3}^{\pi/3} = \frac{1}{2}\left(\frac{\pi}{3} - \frac{-\pi}{3}\right) = \frac{1}{2}\cdot\frac{2\pi}{3} = \frac{\pi}{3}$$

$$\frac{1}{2}\int_{-\pi/3}^{\pi/3}(2\cos\theta)^2\,d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3}4\cos^2\theta\,d\theta = 2\int_{-\pi/3}^{\pi/3}\cos^2\theta\,d\theta = 2\int_{-\pi/3}^{\pi/3}\frac{1+\cos 2\theta}{2}\,d\theta$$
$$= \int_{-\pi/3}^{\pi/3}(1+\cos 2\theta)\,d\theta$$
$$= \left[\theta + \frac{\sin 2\theta}{2}\right]_{-\pi/3}^{\pi/3}$$
$$= \left(\frac{\pi}{3} + \frac{\sin(2\pi/3)}{2}\right) - \left(-\frac{\pi}{3} + \frac{\sin(-2\pi/3)}{2}\right)$$
$$= \frac{\pi}{3} + \frac{\sqrt{3}}{4} + \frac{\pi}{3} + \frac{\sqrt{3}}{4} = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$$

$$A = \frac{1}{2}\int_{-\pi/3}^{\pi/3}(2\cos\theta)^2\,d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3}1^2\,d\theta = \left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right) - \frac{\pi}{3} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

Bonus 1 What is the sum of the following series: $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Bonus 2 What is the name of the following series: $\sum_{n=1}^{\infty} \frac{1}{n}$

The harmonic series.

Bonus 3 Does the following series diverge or converge? Why? [Hint: What grows faster—polynomials or logs?]

$$\sum \frac{1}{\ln(\ln(\ln(n)))}$$

Note that $n > \ln n > \ln(\ln n) > \ln(\ln(\ln(n)))$ *so that* $\frac{1}{n} < \frac{1}{\ln n} < \frac{1}{\ln(\ln n)} < \frac{1}{\ln(\ln(\ln(n)))}$. *But then*

$$\sum \frac{1}{n} < \sum \frac{1}{\ln n} < \sum \frac{1}{\ln(\ln n)} < \sum \frac{1}{\ln(\ln(\ln(n)))}$$

But the harmonic series diverges, so that by the Comparison Test, the given series diverges.