Math 194: Exam 1 **Name:** *Caleb McWhorter — Solutions*
Summer – 2015
07/16/2015
80 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 9 pages (including this cover page) and 7 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	7	
2	11	
3	8	
4	6	
5	11	
6	7	
7	10	
Total:	60	

1. Complete the following parts:

(a) (1 point) Define domain:

The set of all possible inputs to a function, e.g. the "stuff" you can plug into the function.

(b) (1 point) Define range:

The set of all possible outputs to a function, e.g. the "stuff" you can get out of a function.

(c) (1 point) Define linear function:

A function whose graph is a straight line or a function of the form $f(\mathbf{x}) = \mathbf{b} + \mathbf{mx}$.

(d) (3 points) What is the domain of the function $f(x) = \dfrac{\sqrt{x+1}}{x(x-1)}$

The $\sqrt{x+1}$ *portion implies that* $x + 1 \geq 0$*, which means* $x \geq -1$*. The fraction* $\dfrac{1}{x(x-1)}$ *implies that* $x(x-1) \neq 0$ *so that* $x \neq 0$ *or* $x - 1 \neq 0$ *so that* $x \neq 1$*. Therefore, the domain is the set of all* x*'s with* $x \geq -1$ *except for* $x = 0, 1$*, i.e.* $[-1, 0) \cup (0, 1) \cup (1, \infty)$.

(e) (1 point) Is $g(x) = \sqrt{x}$ the inverse to the function $f(x) = x^2$?

No. Observe $g(f(-2)) = g(4) = 2 \neq -2$*. We have* $g(f(x)) = |x|$*, not* $g(f(x)) = x$*. However, it is true that* $f(g(x)) = x$.

2. Let $f(x) = 5x + 14$ and $g(x) = 2x^2 - x - 6$

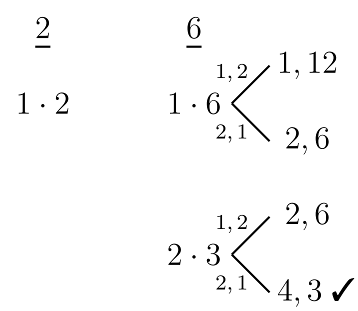
(a) (1 point) What is $g(-1)$?

$$g(-1) = 2(-1)^2 - (-1) - 6 = 2(1) + 1 - 6 = -3$$

(b) (1 point) What is $f(g(-1))$?

$$f(g(-1)) = f(2(-1)^2 - (-1) - 6) = f(-3) = 5(-3) + 14 = -1$$

(c) (4 points) Factor $g(x)$.

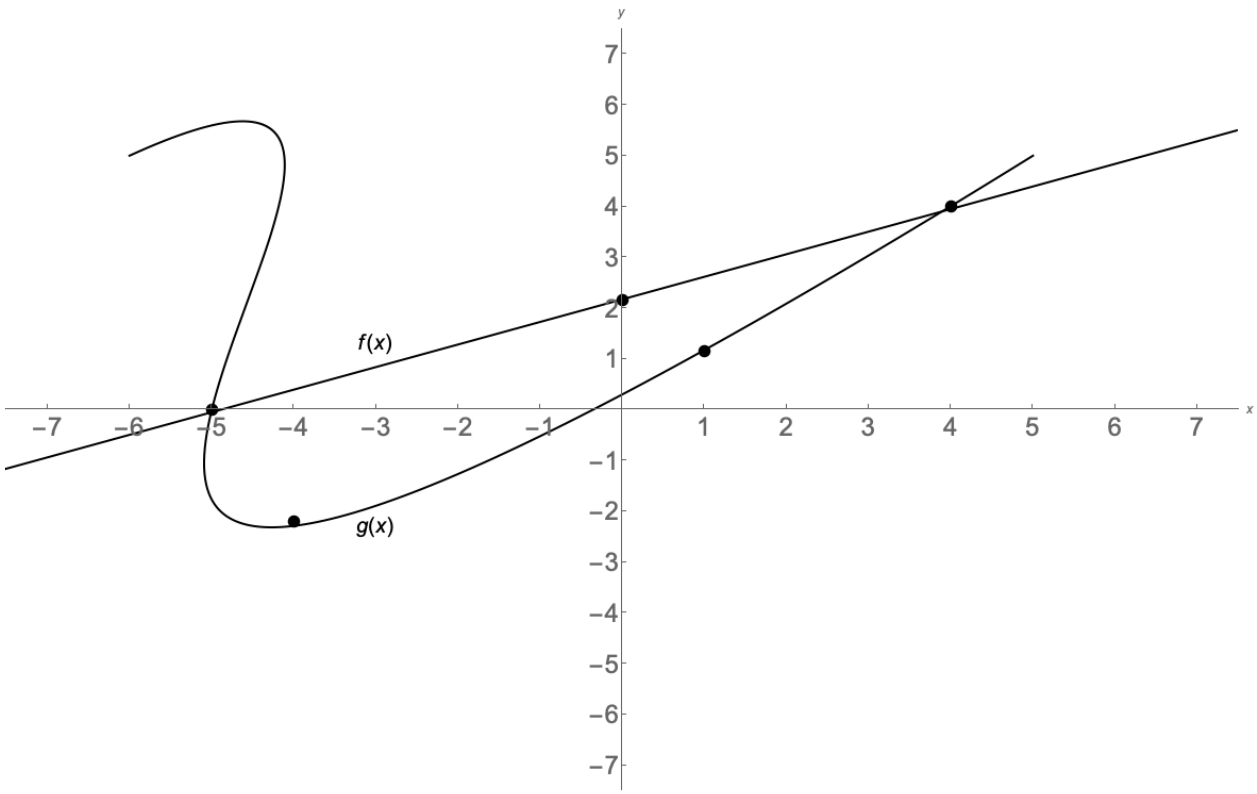


Therefore, $g(x) = (x - 2)(2x + 3)$.

(d) (5 points) Find the points of intersection between $f(x)$ and $g(x)$.

$$\begin{aligned} f(x) &= g(x) \\ 5x + 14 &= 2x^2 - x - 6 \\ 2x^2 - 6x - 20 &= 0 \\ x^2 - 3x - 10 &= 0 \\ (x - 5)(x + 2) &= 0 \end{aligned}$$

Then $x - 5 = 0$ *so that* $x = 5$ *or* $x + 2 = 0$ *so that* $x = -2$. *Now* $f(5) = 5(5) + 14 = 25 + 14 = 39$ *and* $f(-2) = 5(-2) + 14 = -10 + 14 = 4$. *Therefore,* $f(x)$ *and* $g(x)$ *intersect at the points* $(-2, 4)$ *and* $(5, 39)$.



3. Use the graph above to answer the following questions:

(a) (1 point) Is $f(x)$ a function?

Yes, $f(x)$ passes the vertical line test.

(b) (1 point) Is $g(x)$ a function?

No, $g(x)$ fails the vertical line test, e.g. at $x = -5$.

(c) (1 point) What is $g(-4)$?

$g(-4) \approx -2$

(d) (1 point) Give a value of x for which $g(x) = 1$.

$g(1) = 1$ so that $x = 1$ is possible. Notice also that if $x \approx -5$ then $g(-5) \approx 1$.

(e) (2 points) What is the x-intercept for $f(x)$? What is the y-intercept for $f(x)$?

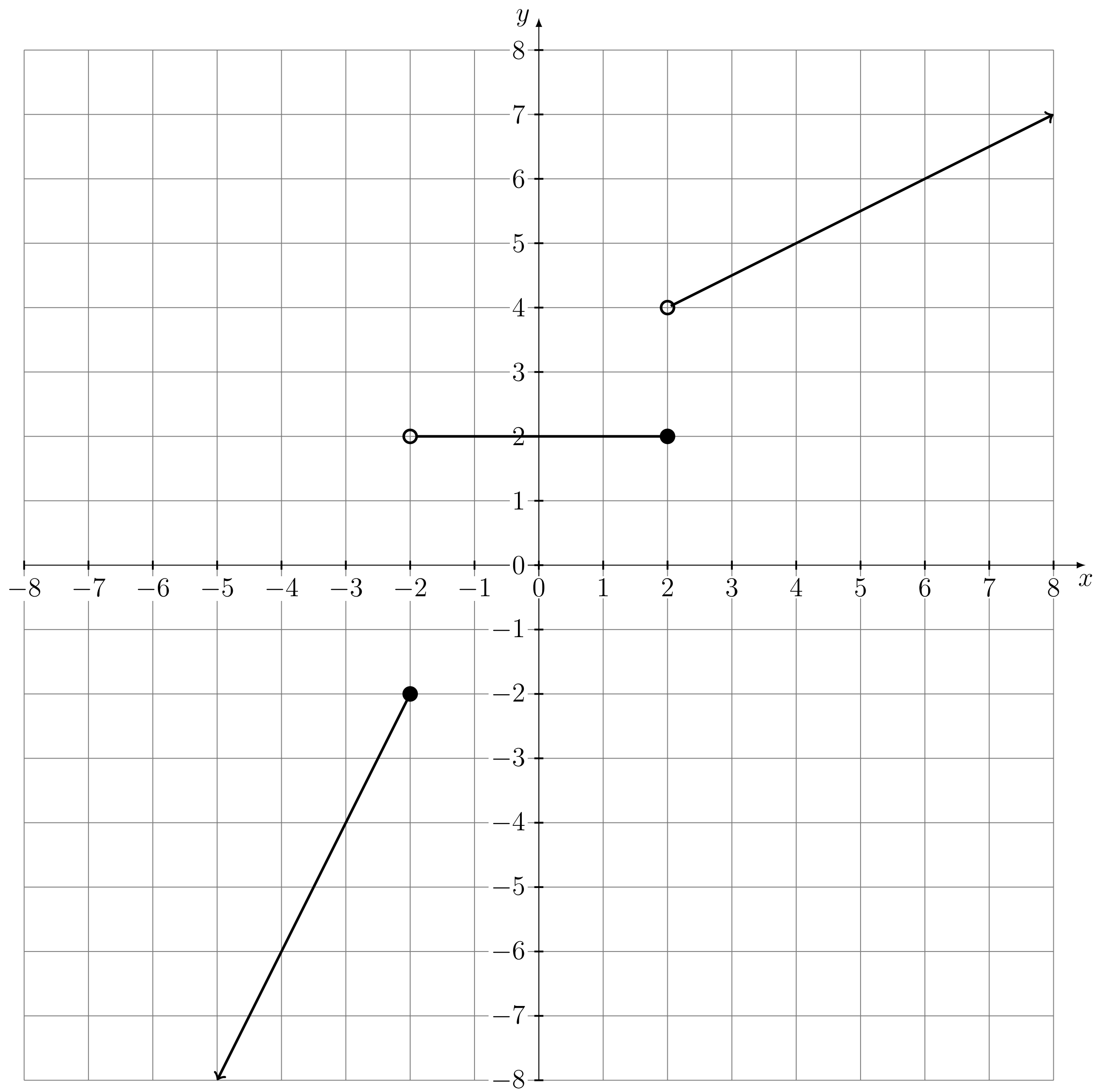
The x-intercepts are $x = -5$, i.e. $(-5, 0)$. The y-intercepts are $y = 2$, i.e. $(0, 2)$.

(f) (2 points) Solve $f(x) = g(x)$ for x.

We have $f(x) = g(x)$ at $x = -5$ and $x = 4$.

4. (6 points) Use the coordinate plane below to graph the function

$$f(x) = \begin{cases} 2x+2, & x \leq -2 \\ 2, & -2 < x \leq 2 \\ \frac{1}{2}x+3, & 2 < x \end{cases}$$



5. Let $f(x) = 3x^2 - 12x + 6$.

(a) (4 points) Find the vertex and axis of symmetry for $f(x)$. Is the vertex a maximum or a minimum?

$x = \frac{-b}{2a} = \frac{-(-12)}{2(3)} = \frac{12}{6} = 2$ *is the axis of symmetry. Now* $f(2) = 3(2^2) - 12(2) + 6 = 3(4) - 24 + 6 = -6$ *so the vertex is* $(2, -6)$. *It must be a minimum as* $a > 0$ *so that the parabola opens upwards.*

(b) (5 points) Use the quadratic equation to solve the equation $f(x) = 0$.

We know $72 = 2{\cdot}36 = 2{\cdot}6{\cdot}6 = 2{\cdot}2{\cdot}3{\cdot}2{\cdot}3 = 2^3{\cdot}3^2$. *But then* $\sqrt{72} = \sqrt{2\cdot 2\cdot 2\cdot 3\cdot 3} = \sqrt{2\cdot 2\cdot 3\cdot 2}\sqrt{2} = 2\cdot 3\sqrt{2} = 6\sqrt{2}$. *We know* $f(x) = 3x^2 - 12x + 6$ *so that* $a = 3$, $b = -12$, *and* $c = 6$. *Then*

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)6}}{2(3)} \\ &= \frac{12 \pm \sqrt{144 - 72}}{6} \\ &= \frac{12 \pm \sqrt{72}}{6} \\ &= \frac{12 \pm 6\sqrt{2}}{6} \\ &= 2 \pm \sqrt{2} \end{aligned}$$

(c) (2 points) Use the previous part to give the factorization for $f(x)$.

$$f(x) = 3x^2 - 12x + 6 = 3(x^2 - 4x + 2) = 3\Big(x - (2 + \sqrt{2})\Big)\Big(x - (2 - \sqrt{2})\Big)$$

6. Complete the following parts:

(a) (2 points) Find the equation of a line with slope 3 and y-intercept -5.

$y = mx + b$ *so that* $y = 3x - 5$

(b) (3 points) Find the equation of a line passing through the points $(-3, 2)$ and $(1, 8)$.

$m = \dfrac{8-2}{1-(-3)} = \dfrac{8-2}{1+3} = \dfrac{6}{4} = \dfrac{3}{2}$. *We know that* $y = mx + b$ *and that the point* $(1, 8)$ *is on the line so that*

$$\begin{aligned} y &= mx + b \\ y &= \frac{3}{2}x + b \\ 8 &= \frac{3}{2} \cdot 1 + b \\ 8 &= b + \frac{3}{2} \\ b &= 8 - \frac{3}{2} = \frac{16}{2} - \frac{3}{2} = \frac{13}{2} \end{aligned}$$

Therefore, $y = \dfrac{3}{2}x + \dfrac{13}{2} = \dfrac{3x+13}{2}$.

(c) (2 points) Find an equation for a parabola with zeros $x = -4$ and $x = 5$.

From the given information, we know

$$f(x) = a(x - (-4))(x - 5) = a(x + 4)(x - 5)$$

Now a *can be any nonzero number so choosing* $a = 1$*, we have* $f(x) = (x + 4)(x - 5) = x^2 - x - 20$ *is a possible parabola.*

7. Paul and Mary both open a checking account on the same day. The amount of money in euros in Paul's checking account t months after opening it is given by $P(t) = 2700 - 60t$ while the amount of money in euros in Mary's account t months after opening it is given by $M(t) = 3400 - 70t$.

   (a) (2 points) What is the y-intercept for $M(t)$? What does it represent in this context?

   We know the y-intercept is $y = 3400$, i.e. $(0, 3400)$. This represents that Mary had \$3,400 deposited into the account when it was created.

   (b) (2 points) What is the x-intercept for $P(t)$? What does it represent in this context?

   We know $2700 - 60t = 0$ so that $60t = 2700$ which means $t = 45$. Then the x-intercept is $x = 45$, i.e. $(45, 0)$. This represents that Paul's account was out of money after 45 months (almost 4 years).

   (c) (2 points) What is the slope for $M(t)$? What does it represent in this context?

   The slope is $-70 = \dfrac{-70}{1}$. This means there is a net loss of \$70 from the account every month.

   (d) (4 points) When do Paul and Mary have the same amount of money in their respective checking accounts? How much money do they have when they have the same amount?

$$\begin{aligned} P(t) &= M(t) \\ 2700 - 60t &= 3400 - 70t \\ 10t &= 700 \\ t &= 70 \end{aligned}$$

   We know $P(70) = 2700 - 60(70) = 2700 - 4200 = -\$1500$. Therefore, they have the same amount of money after 70 months. They each have $-\$1,500$ in the account, i.e. they both owe \$1,500.

Bonus Problems

The following are bonus questions. You should not attempt these questions until you are content with your responses on all other parts of the exam.

Bonus 1: Complete the following table:

Name	Polynomial Degree	Minimum Number of Points to [Uniquely] Determine
Constant	*0*	*1*
Linear	*1*	*2*
Quadratic	*2*	*3*
Cubic	*3*	*4*
Quartic	*4*	*5*
Quintic	*5*	*6*

Bonus 2: Determine the equation of *any* parabola that passes through the points $(-2,-1)$ and $(3,1)$. Show how you arrived at this parabola.

We know that $y = f(x) = ax^2 + bx + c$*. Then*

$$-1 = a(-2)^2 - 2b + c \Rightarrow -1 = 4a - 2b + c$$
$$1 = a(3)^2 - 3b + c \Rightarrow 1 = 9a - 3b + c$$

We have too few points to uniquely determine. So we will have to choose a variable, as long as $a \neq 0$ *because otherwise the function is not a parabola and is instead a line. We choose* $a = 1$ *and see if this yields a solution. Then plugging this into* $-1 = 4a - 2b + c$ *and* $1 = 9a - 3b + c$ *gives*

$$-5 = -2b + c$$
$$-8 = -3b + c$$

Subtracting the equations yields $b = 3$*. But then* $-5 = -2(3) + c$ *so that* $c - 6 = -5$ *which implies* $c = 1$*. Therefore, a possible parabola is* $f(x) = x^2 + 3x + 1$*.*