Math 194: Exam 2
Summer – 2015
07/29/2015
80 Minutes

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Write your name on the appropriate line on the exam cover sheet. This exam contains 9 pages (including this cover page) and 7 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	9	
3	9	
4	9	
5	11	
6	9	
7	9	
Total:	66	

1. Complete the following parts:

   (a) (1 point) Define exponential function:

   A function of the form $f(x) = Ar^x$.

   (b) (3 points) Simplify the following:

$$\ln\left(\sqrt{\frac{e^9e^{-3}}{e^2}}\right)$$

$$\ln\sqrt{\frac{e^9e^{-3}}{e^2}} = \ln\sqrt{\frac{e^6}{e^2}} = \ln\sqrt{e^4} = \ln e^2 = 2$$

   (c) (3 points) Simplify the following:

$$\frac{\log_5\left(\frac{1}{5}\right)}{\log_5(5)} + \frac{1}{2}\log_5(25) + \log_5(1)$$

$$\frac{-1}{1} + \frac{1}{2}(2) + 0 = -1 + 1 + 0 = 0$$

   (d) (3 points) Simplify the following:

$$2e^{\ln(x-1)+\ln(x+1)}$$

$$\begin{array}{lcl} 2e^{\ln((x-1)(x+1))} & \textit{or} & 2e^{\ln(x+1)}e^{\ln(x+1)} \\ 2(x-1)(x+1) & & 2(x-1)(x+1) \\ 2(x^2-1) & & 2(x^2-1) \\ 2x^2-2 & & 2x^2-2 \end{array}$$

2. Complete the following parts:

(a) (1 point) Write the equation $10^x = 3$ in terms of logs.

$$10^x = 3 \iff x = \log_{10} 3$$

(b) (1 point) Write the equation $\log_5 4 = x$ in terms of exponential functions.

$$\log_5 4 = x \iff 5^x = 4$$

(c) (3 points) If x, y are numbers such that $2^x = 256$ and $\log_2 y = 3$, does $x = y$? Be sure to show why or why not.

$2^x = 256 \iff x = \log_2 256 \iff x = 8$ *and* $\log_2 y = 3 \iff y = 2^3 \iff y = 8$. *Then yes,* $x = y$.

(d) (4 points) Find the equation of an exponential function that goes through the points $(0, 100)$ and $(2, 4)$.

We have $y = ab^x$. *Using* $(0, 100)$*, we know* $100 = ab^0 = a$. *Therefore,* $y = 100b^x$. *Using* $(2, 4)$*, we know that* $4 = 100b^2$ *so that* $b^2 = \frac{1}{25}$. *Therefore,* $b = \frac{1}{5}$. *Then*

$$y = 100\left(\frac{1}{5}\right)^x$$

3. Solve for x in the following equations:

(a) (3 points)

$$x^2 4^x - 4^x = 0$$

$$\begin{aligned} x^2 4^x - 4^x &= 0 \\ 4^x(x^2 - 1) &= 0 \\ 4^x(x-1)(x+1) &= 0 \end{aligned}$$

Therefore, $4^x = 0$ *or* $x - 1 = 0$ *or* $x + 1 = 0$*. The first equation produces no solution but the last two give the solutions* $x = 1$ *or* $x = -1$*, respectively. Therefore,* $x = \pm 1$*.*

(b) (3 points)

$$\log_3(9x) - \log_3(x) = \log_5(x-1)$$

$$\begin{aligned} \log_3\left(\frac{9x}{x}\right) &= \log_5(x-1) \\ \log_3 9 &= \log_5(x-1) \\ 2 &= \log_5(x-1) \\ 5^2 &= x - 1 \\ 25 &= x - 1 \\ x &= 26 \end{aligned}$$

(c) (3 points)

$$e^{2x} - 5e^x + 6 = 0$$

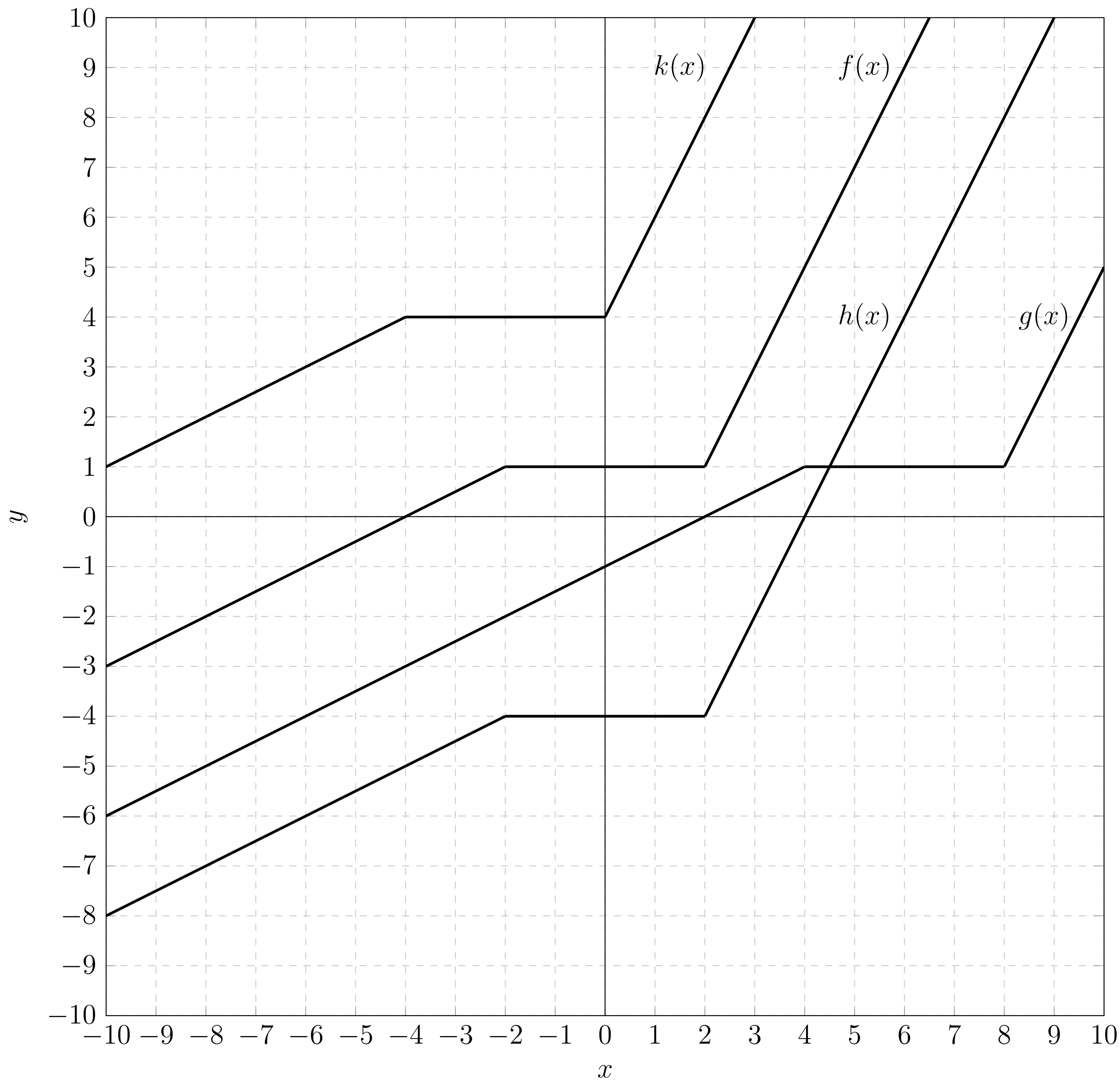
Let $e^x = y$*. Then*

$$\begin{aligned} y^2 - 5y + 6 &= 0 \\ (y-2)(y-3) &= 0 \end{aligned}$$

But then $y - 2 = 0$ *or* $y - 3 = 0$*. The first implies that* $y = 2$ *so that* $e^x = 2 \iff x = \ln 2$*. The second implies* $y = 3$ *so that* $e^x = 3 \iff x = \ln 3$*.*

4. (9 points) The following piecewise function is graphed below.

$$f(x) = \begin{cases} \frac{1}{2}x + 2, & x \leq -2 \\ 1, & -2 < x \leq 2 \\ 2x - 3, & 2 < x \end{cases}$$



Using the graph above, carefully graph and label the following functions:

(i) $g(x) \stackrel{\text{def}}{=} f(x-6)$

(ii) $h(x) \stackrel{\text{def}}{=} f(x) - 5$

(iii) $k(x) \stackrel{\text{def}}{=} f(x+2) + 3$

5. Complete the following parts:

   (a) (2 points) If $f(x)$ is a function. What does the graph of $g(x) \stackrel{\text{def}}{=} f(x-1)+7$ look like compared to the graph of $f(x)$?

   The graph looks like the graph of $f(x)$ shifted one unit to the right and seven units upward.

   (b) (2 points) What is the growth/shrink rate of $r(s) = 15(1.15)^s$ and $q(n) = 17(0.98)^n$?

   $r(s)$ *has growth rate* 15% *while* $q(n)$ *has shrink rate* 2%.

   (c) (4 points) What is the growth/shrink rate of $y(z) = 23(1.03)^{2z}$ and $k(u) = 9(0.92)^{-u}$?

   $y(z) = 23((1.03)^2)^z = 23(1.0609)^z$ *has growth rate* 6.09%. $k(u) = 9((0.92)^{-1})^u = 9(1.08696)^u$ *has growth rate* 8.696%.

   (d) (3 points) If $x = \log A$ and $y = \log B$, express $\log\left(\sqrt{\frac{A^4}{B^2}}\right)$ in terms of x, y.

$$\log\sqrt{\frac{A^4}{B^2}} = \log\frac{A^2}{B} = \log A^2 - \log B = 2\log A - \log B = 2x - y$$

6. Complete the following parts:

(a) (3 points) How many digits does $155^{17,142,546}$ have?

$\log_{10} 155^{17142456} = 17142456 \log_{10} 55 = 37547861.89$. *Therefore,* $155^{17,142,546}$ *has 37,547,862 digits.*

(b) (3 points) If $\ln(\log_7 x) = 12$, how many digits does x have?

First, $\ln(\log_7 x) = 12 \iff \log_7 x = e^{12} \iff x = 7^{e^{12}}$. *Then* $\log_{10} x = \log_{10} 7^{e^{12}} = e^{12} \log_{10} 7 = 137543.75$. *Therefore,* x *has* $137,544$ *digits.*

(c) (3 points) Find an integer k such that 17^k has 1830 digits.

17^k *has* $\lceil \log_{10} 17^k \rceil$ *digits. So we want*

$$\begin{aligned} 1829 &\leq \log_{10} 7^k < 1830 \\ 1829 &\leq k \log_{10} 17 < 1830 \\ \frac{1829}{\log_{10} 7} &\leq k < \frac{1830}{\log_{10} 17} \\ 1486.4 &\leq k < 1487.2 \end{aligned}$$

Therefore, $k = 1,487$ *is such an integer.*

7. (9 points) Choose ***one*** of the following problems and complete it in its entirety. Place a checkmark next to the number of the problem you have chosen.

   (i) A bank account is opened with an initial deposit of \$3000. The account has a monthly compounded interest rate of 6%. Write a function, $M(t)$, representing the amount of money in the account t years after it was opened. How much is in the account after 2 years? When does the account have \$4600?

   (ii) A certain radioactive substance has a half-life time of 2.7 years. If 287kg of this substance is stored away, write a function, $A(t)$, representing the amount of this radioactive substance remaining after t years. How much of this substance is left after 2 years? When will only 129kg of this substance remain?

(i)

(a)

$$\begin{aligned} M(t) &= 3000\left(1+\frac{0.06}{12}\right)^{12t} \\ &= 3000(10.005)^{12t} \\ &= 3000(1.0616778)^{t} \end{aligned}$$

(b) $M(2) = \$3,381.48$

(c)

$$\begin{aligned} M(t) &= 4600 \\ 3000(1.0616778)^t &= 4600 \\ 1.0616778^t &= \frac{4600}{3000} \\ \ln(1.0616778^t) &= \ln\left(\frac{4600}{3000}\right) \\ t\ln(1.0616778) &= \ln\left(\frac{4600}{3000}\right) \\ t &= \frac{\ln\left(\frac{4600}{3000}\right)}{\ln(1.0616778)} \\ t &\approx 7.14 \textit{ years} \end{aligned}$$

(ii)

(a) $A(t) = 287\left(\frac{1}{2}\right)^{t/2.7}$

(b) $A(2) = 171.75$ *kg*

(c)

$$\begin{aligned} A(t) &= 129 \\ 287\left(\frac{1}{2}\right)^{t/2.7} &= 129 \\ \left(\frac{1}{2}\right)^{t/2.7} &= \frac{129}{287} \\ \ln\left(\frac{1}{2}\right)^{t/2.7} &= \ln\left(\frac{129}{287}\right) \\ \frac{t}{2.7}\ln\left(\frac{1}{2}\right) &= \ln\left(\frac{129}{287}\right) \\ t &= 2.7\frac{\ln\left(\frac{129}{287}\right)}{\ln\left(\frac{1}{2}\right)} \\ t &\approx 3.11 \textit{ years} \end{aligned}$$

Bonus

The following are bonus questions. You should ***not*** attempt these bonus questions until you are content with your responses on all other parts of the exam.

(i) (2 points) Write the formula for the amount of money, $M(t)$, in a bank account after t years if it began with \$4000 and has a 4.6% interest rate compounded continuously.

$$M(t) = 4000e^{0.046t}$$

(ii) (10 points) Solve the equation:

$$\ln(x)^2 + \ln\left(\frac{1}{x^3}\right) + 1 = 0$$

Let $y = \ln x$*, then*

$$\begin{aligned}
\ln(x)^2 + \ln\left(\frac{1}{x^3}\right) + 1 &= 0 \\
(\ln x)^2 + \ln(x^{-3}) + 1 &= 0 \\
(\ln x)^2 - 3\ln x + 1 &= 0 \\
y^2 - 3y + 1 &= 0 \\
y &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)1}}{2(1)} \\
y &= \frac{3 \pm \sqrt{9-4}}{2} \\
y &= \frac{3 \pm \sqrt{5}}{2} \\
\ln x &= \frac{3 \pm \sqrt{5}}{2} \\
x &= e^{\frac{3 \pm \sqrt{5}}{2}}
\end{aligned}$$