Linear Problems

Problem 1: Write a relationship using function notation between the following:

(i) The temperature in a room is a function of the number of heating vents in the room.

(ii) Calories burned in the gym is a function of the length of time spent in the gym.

(iii) Time is a function of velocity.

(iv) Most likely native language is a function of your hometowns location.

(v) Your disposable income is a function of your income, debts, bills, and number of children.

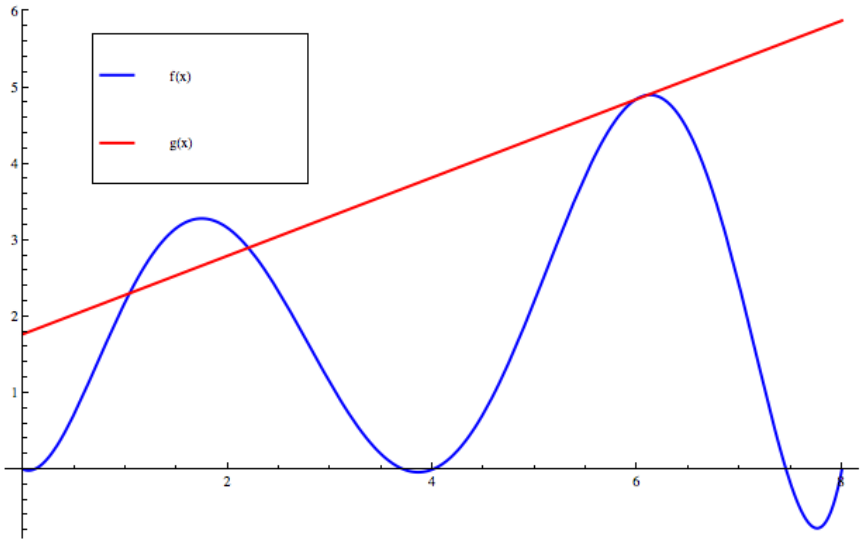
Problem 2: Is the area of a square a function of the length of its side? Is the area of a rectangle a function of the length of one of its sides? What about both?

Problem 3: The sales tax on an item is 6%. Express the total cost, C, in terms of the price of the item, P.

Problem 4: Write down the formula for the area of a circle. Determine the formula for the area of this circle if the radius is increased by 10%. What is the percent increase in area?

Problem 5: Sketch a graph to model the following situation: a piece of anchoring metal for a bridge is cool in the morning until the sun rises but then heats from sunlight. But then the sun rises high enough for a tree shadow to cover the metal causing it to cool slightly then the sunlight touches the metal again and it heats until the temperature is constant. Then midday comes and the sun sets and the metal cools.

Problem 6: Use the following plot for the parts below:



(i) What is $f(5)$?

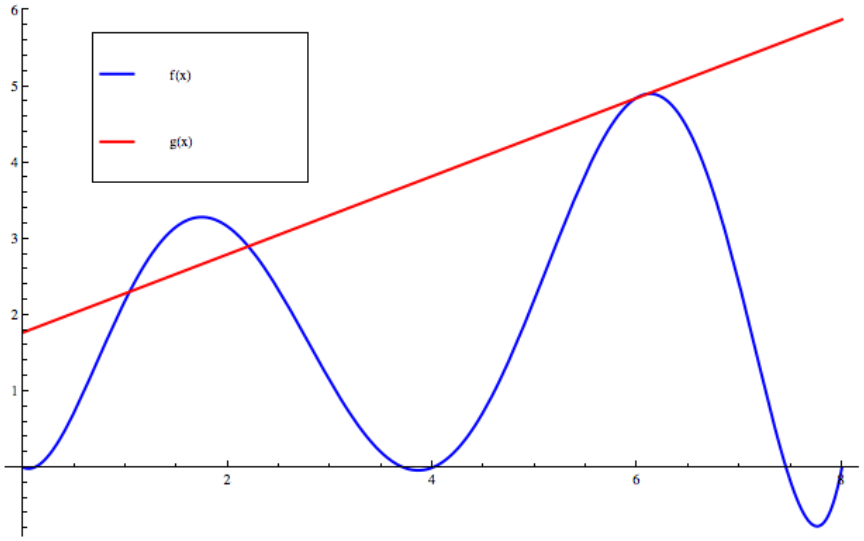
(ii) Give a point on the graph $f(x)$ and two points on the graph of $g(x)$.

(iii) Solve $f(x) = 0$.

(iv) Solve $f(x) = g(x)$.

(v) When is $f(x)$ increasing? When is $f(x)$ decreasing? When is $g(x)$ increasing?

Problem 7: Using the graph below, find the average rate of change of $f(x)$ between 2 and 4, 0 and 2, and 0 and 8. Determine the average rate of change for $g(x)$ on the interval $[2, 6]$. Would this be the same as for the interval $[0, 8]$? Give two intervals on which the average rate of change for $f(x)$ is 0.



Problem 8: If $\varphi(x)$ is an increasing function, what could you say about $\varphi(3) - \varphi(-1)$?

Problem 9: If $\zeta(x)$ is a decreasing function, what is the sign of $\zeta(0) - \zeta(-1)$?

Problem 10: Simplify the following:

$$\frac{x^2 - \frac{3}{4} - (y^2 - \frac{3}{4})}{x - y}$$

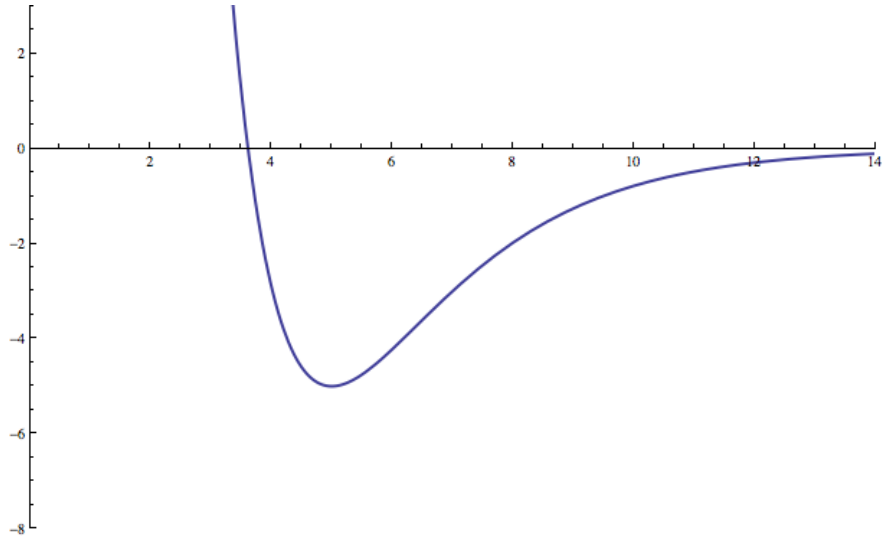
Problem 11: Simplify the following:

$$x^2 - (2x + a)^2$$

Problem 12: Simplify the following:

$$2(x + a) - 3(x - b)$$

Problem 13: Use the following graph for the parts below:



(i) Estimate $\frac{f(8)-f(5)}{8-5}$.

(ii) The ratio in the previous part is the slope of what? Indicate what by drawing it in on the graph.

Problem 14: Are the following linear functions?

x	0	100	300	600
$g(x)$	50	100	150	200

t	1	2	3	4
$\eta(t)$	5	4	5	

y	0	5	10	15
$h(y)$	9	8	7	6

r	-2	-1	0	3
$\gamma(r)$	5	1	-1	-7

Problem 15: Identify the vertical intercept and slope of the following linear functions. Give these physical meaning:

(i) A stalactite grows according to $L(t) = 17.75 + \frac{1}{250}t$, where $L(t)$ is its length in inches and t is time in years.

(ii) The profit, in dollars, of selling n items is given by $P(n) = 0.98n - 3000$.

(iii) A phone company charges according to the formula $C(n) = 29.99 + 0.05n$, where n is the number of minutes and $C(n)$ is the monthly phone charge in dollars.

Problem 16: Determine the equation of the linear function:

(i) Slope 3 y-intercept 8.

(ii) Slope -4 and x-intercept 7.

(iii) Slope $2/3$ and passes through the point $(-1, 5)$ and $(2, -1)$.

(iv) Passes through the point $(1, 3)$ and $(5, 19)$.

(v) Such that $f(1) = -1$ and $f(-1) = 1$.

(vi) x-intercept 3 and y-intercept of 5

Problem 17: Solve the following equation for x: $ab + ax = c - ax$.

Problem 18: Solve the following system:

$$x + y = 3$$
$$x - y = 5$$

Problem 19: Solve the following system:

$$3x - 5y = -7$$
$$4x + 3y = 10$$

Problem 20: Determine the point of intersection (if it exists) for the lines $y = \frac{-1}{2}x + 4$ and $y = 2x - 3.5$.

Problem 21: Are the following lines parallel: $y = 5x - 7$ and $y = 5x + 8$? Why or why not? Are the lines $y = 4x + 3$ and $y = 13 - \frac{1}{4}x$ parallel? Do these two lines have anything special about them?

Problem 22: What is true about the constant β in the following linear equation if its graph has the given property:

$$y = \frac{x}{\beta - 3} + \frac{1}{6 - \beta}$$

(i) Positive slope, positive y-intercept

(ii) Perpendicular to the line $y = (\beta - 7)x - 3$.