Exponential & Logarithmic Problems

Problem 1: Because of a business owner's constant withdrawals, their bank account loses 6% of its total each year. By what total percent does it decrease after 4 years? Find a formula $M(t)$ to represent the amount in the account after t years.

Problem 2: Let $P(t)$ denote the population of some city t years from today.

(i) $P(t) = 1000(1.08)^t$

(ii) $P(t) = 2500(0.9)^t$

(iii) $P(t) = 800(0.78)^t$

(iv) $P(t) = 600(1.12)^t$

(v) $P(t) = 1200(1.185)^t$

(vi) $P(t) = 2000(0.99)^t$

Of the cities above, which are growing in size and which are shrinking? Which town is growing the fastest? Which is growing the slowest? Which town had the greatest population initially? Which had the least?

Problem 3: Write how you calculate the following:

(i) 27% of 12

(ii) 117% of 283

(iii) 100% of 0.12

Problem 4: Write a formula for the following situations:

(i) Initial Amount: 150, Increasing by 4% a year

(ii) Initial Amount: 132, Decreasing by 36% a year

(iii) Initial Amount: 164, Increasing by 45% a year

(iv) Initial Amount: 124, Increasing by 234% a year

Problem 5: Forty percent of a radioactive substance decays in five years. By what percent does the substance decay each year?

Problem 6: Give the growth rate for the exponential function $Y(t) = 12.32(2.15)^{-3t}$ and $G(t) = 17.21(1.12)^{2t}$.

Problem 7: Write the equation $f(t) = 1000 \cdot 16^{-\frac{1}{4}-\frac{t}{2}}$ in the form $f(t) = ab^t$.

Problem 8: Find a formula for an exponential function such that $f(2) = \frac{-1}{27}$ and $f(-1) = -27$.

Problem 9: If $M(t) = 500\left(1 + \frac{0.04}{12}\right)^{12t}$ represents the amount of money in an account after t years, describe what the formula represents, i.e. what is the initial amount and by what percent is the money growing/shrinking each year?

Problem 10: Write an equation representing the value of an account starting at \$5,000 if the account has an interest rate of 7% and the interest is compounded biyearly. What if the account had an interest rate of 4% compounding continuously?

Problem 11: What interest rate is necessary if an account that has interest compounded continuously is to always have the same amount of money as an account that has interest compounded monthly—assuming the accounts start with the same amount of money.

Problem 12: Simplify the following:

(i) $\ln e^{2x}$

(ii) $\ln\left(\frac{1}{e^{5x}}\right)$

(iii) $e^{\ln(3x+2)}$

(iv) $e^{\ln 3x+2}$

(v) $\ln\sqrt{e^x}$

(vi) $\ln \frac{1}{\sqrt{e}}$

(vii) $\ln e^0$

Problem 13: Solve for x in the following:

(i) $e^{x+4} = 10$

(ii) $\log_6(x-2) = 2$

(iii) $\log_4(3x+1) + 1 = 3$

(iv) $\ln(\log_7(x)) = 2$

(v) $\ln(x+1) - \ln(x-1) = 3$

(vi) $\ln\left(\frac{1}{x}\right) = 3$

(vii) $e^{x+5} = 7 \cdot 2^x$

(viii) $\log_3 2 + \log_3 x = \log_6 36 + \log_3(x+3)$

Problem 14: Calculate how long it will take for the population of a city to double or triple if the city grows at a constant rate of

(i) 1000 citizens per year

(ii) 4% per year

Problem 15: The temperature of a basic $\$63.99$ Sawbucks cup of coffee is given by

$$H(t) = 70 + 120\left(\frac{1}{4}\right)^t$$

How hot is the coffee initially? How hot is it after 3 hours? How long does it take for the coffee to reach 75°F?

Problem 16: Solve for x in the following:

(i) $e^{2x} + 2e^x - 3 = 0$

(ii) $x^2 3^x - 4 3^x = 0$

(iii) $\frac{5}{1+e^{-x}} = 1$

(iv) $e^x - 20e^{-x} = 1$

Problem 17: Show every exponential function can be put in terms of ae^{kt} for some a, k.

Problem 18: If $x = \log A$ and $y = \log B$, then what is $\log\left(\frac{A}{B^2}\right)$?

Problem 19: How many digits does 6^{12345} have?

Problem 20: Find an integer b such that $\log_b f$ has slope 1, where $f(x) = 7(2^{3x})$.

Problem 21: Show that the square root of an exponential function is still an exponential function.

Problem 22: Find an integer k such that 13^k has 1729 digits.

Problem 23: If $\ln(\log_6 x) = 7$, how many digits does x have?