Final Project

Mathematics has beauty and romance. It's not a boring place to be, the mathematical world. It's an extraordinary place; it's worth spending time there.

$\sim$ Marcus du Sautoy

Guidelines

For this project, you may work individually or in groups. This project is in two parts:

First, every individual/group should have a sheet defining the following terms:

- Conjecture
- Theorem
- Proof
- Corollary
- Lemma

This sheet should also list as many fields of Mathematics as possible with a brief description of what types of problems people in these field work on.

Second, choose a topic from the list below. Once you choose a topic, choose two or three subtopics from the theme and research them. The point is not to be an expert at the topics but rather to be able to explain the general idea—even in the most abstract form—to the class. You will summarize what you learned, your impressions, or any other things that came to mind during your research in an approximately one page, size 12 font, single spaced, typed paper.

During the last week of class, you will present whatever you would like about what you researched to the class in a short 5–15 min presentation. Should you choose to work in groups, each group member is expected to contribute to the presentation and submit their own paper about what they learned. Each group member need not research the same subtopics or write about the same things in your paper, but you must agree on what to present to the class.

You must set up an appointment with me at least a week before your presentation to discuss what you will be talking about for approval. The meeting need not last long but should give me an idea of what you will be discussing. You may feel free at any point to ask me questions if you are confused about any of the mathematics in your topic—this project is meant to challenge you! There is opportunity for extra credit in the course for projects which are especially well done.

Topics

Games Game Theory is a field of Mathematics which looks at strategic decision making. The theory began with deep mathematical ideas but quickly spread to actual games, and these concepts were quickly extended to try to describe stock market behavior, biological functions, how companies behave like organisms or individuals, and even explaining human behavior. One of the most famous game theorists was John Nash, the subject of the film *A Beautiful Mind*. Your job is to explore Game Theory and what it is. What is a game? What are the types of games? What does it mean for a game to be 'solved'? How does Game Theory apply to the real world? Suggested areas of investigation include:

- **Instant Insanity:** This predecessor to the Rubik Cube has $41,472$ states but with how many solutions are there? Can you solve it?
- **Rubik Cube:** This puzzle has dazzled people since 1974 with $43,252,003,274,489,856,000$ possible arrangements. How is it solved? What is the world record for solving the cube? How many moves does it take *at least* to solve the cube? How does this demonstrate the mathematical concept of a *group*? What are other sizes or variations on the Rubik cube?
- **NIM:** A game that has existed in different variations for centuries. Can you always win? How does knowing this game allow you to win at hundreds of other games?
- **Sprouts:** This game can even be played by children but has it been solved? Who invented this game? How do you win for 3 sprouts?
- **Surreal Numbers:** We know that ∞ is not a number. But can it be made into a number so things like $\infty + \infty = 2\infty$? What do these numbers have to do with games?
- **Checkers:** Almost every child has played checkers but what is known about the mathematics of checkers? Can the first player always win? The second? What has computers allowed us to do with checkers? How does the situation compare to Chess?
- **Hakenbush:** Can you play a game with any picture that a child draws for you? What is the strategy?
- **Prisoner's Dilemma:** You are caught along with a friend committing a crime. When you are hauled in for questioning, what should you do when presented with a deal to rat out your friend? How does this appear in the film *A Beautiful Mind* and was it properly applied? How does this relate (nor not relate) to the real world? How does this reflect on human nature?
- **Tic Tac Toe:** Can you always win at a game of Tic Tac Toe? What is the strategy? What is Tic Tac Toe on larger boards or in more dimensions? Is the strategy the same? Are there other more difficult versions of Tic Tac Toe?

Mathematics & the Universe "Most people would find the picture of our universe as an infinite tower of tortoises rather ridiculous, but why do we think we know better? What do we know about the universe, and how do we know it? Where did the universe come from, and where is it going? Did the universe have a beginning, and if so, what happened before then? What is the nature of time? Will it ever come to an end? Can we go back in time? Recent breakthroughs in physics, made possible in part by fantastic new technologies, suggest answers to some of these longstanding questions. Someday these answers may seem as obvious to us as the earth orbiting the sun or perhaps as ridiculous as a tower of tortoises. Only time (whatever that may be) will tell."[1]

- **Mathematica & Wolfram Alpha:** Stephen Wolfram has developed a powerful tool to try to singularize human knowledge and make real world situations and knowledge computable. Using these tools, Wolfram may have found a way to determine the nature of the universe different from that of Quantum Mechanics, String Theory, or any other method of Physics. See his talk here: http://www.ted.com/talks/stephen_wolfram_computing_a_theory_of_everything?language=en You can also see hundreds of freely interactive demonstrations using Mathematica at http://demonstrations.wolfram.com/

- **Cellular Automata:** How can Mathematics replicate the complexity of life? Can we combine this with computers even to replicate not only our world but all possible worlds and discover the nature of the Universe? Learn more by watching Stephen Wolframs talk at http://www.ted.com/talks/stephen_wolfram_computing_a_theory_of_everything?language=en It is recommended you reach "cellular automata" and "Conways game of life" into YouTube to see demonstrations of these or perhaps go to Wolfram Demonstrations to try a few of your own. How do these relate to how you might design a video game—even one you could design such as Minecraft? Perhaps you want to read the section in Richard Dawkin's book *The Blind Watchmaker* on his stick figure programs and examine how these might describe evolutionary processes.

- **Mathematical Universe Hypothesis:** Observe that Mathematics describes, models, and makes predictions about the real world with stunning accuracy. How is this possible? Could our Universe actually *be* Mathematics?

- **Turing Test:** Who is credited with creating the concept of a Turing Test? Describe their life. If you are talking to someone through a computer, how do you know that this "person" is a human? How could you tell if it was a computer? What does this tell you about what it means to be human; what it means to be intelligent; what it means to be alive; what it means even to exist?

- **Chaitin's Omega, Ω** : If you tell a computer to do something, how do you know it will ever finish? What does this have to do with what is known or can be known. It is said of Ω, that Ω "reveals that mathematics. . . is mostly made of gaping holes. Anarchy. . . is at the heart of the universe."[2]

- **$\mathbf{E_8}$** : What can an imagined mathematical object has to do with the nature of the Universe. Does the nature of all things lie in E_8? What does E_8 have to do with a unified theory of physics, string theory, and gravity? Should we believe things are that "simple"?

[1] Hawking, Stephen, 1988: *A Brief History of Time*. Bantam, 212 pp.
[2] Pickover, Clifford, 2009: *The Mαth βooκ*. Sterling, 528 pp.

- **Chaos Theory:** How can small changes lead to vast differences in systems? What does this have to do with the evolutions of systems in the sciences?

Mathematical Paradoxes

Mathematics developed slowly as did our understanding of the natural world. Scientists and Mathematicians stumbled to put meaning to words like time, space, length, sets or things, and in the process many apparent contradictions arose. How do we resolve such breakdowns of logic? Are all these "paradoxes" failure of correct logical definitions or a failure of human intuition? Or are these signs of the progress of Science and Mathematics?

- **Zeno's Paradox:** Is motion an illusion? Is the passage of time a figment of our imagination? This paradox baffled the ancient Greeks, can you resolve it?
- **Monty Hall Problem:** Would you like to try to win a goat or a new car? Would you perhaps care to change your answer? This problem has great explanations on YouTube.
- **Benford's Law:** In my business this week, I made sales this week of $\$415.65$, $\$732.54$, $\$551.98$, $\$813.46$, $\$919.47$, $\$121.13$, and $\$971.17$. Should you believe me?
- **Aristotle's Wheel Paradox:** How can two wheels with the same amount of points trace out two different lengths? That is, how can wheels with different circumferences "trace out" the same length?
- **Barber Paradox:** Suppose the barber shaves everybody in town, except for all of those who shave themselves. Who shaves the barber? How does this relate to Russell's Paradox and why did it shake the foundations of Mathematics?
- **Birthday Paradox:** How many people do you have to have in a room until someone sharing your birthday is a "sure" thing? How do you calculate this?
- **Coastline Paradox:** How does measuring the length of a coastline get "worse" the better you try to measure it? How does this relate to fractals? What are some examples of fractals, both mathematically and in the real world?
- **Newcomb Paradox:** How do you best compete for the biggest prize possible against an angel whose vast predictive powers are working against you?
- **Parrando's Paradox:** How can playing a game guaranteed to make you lose all your money make you rich?
- **Banache-Tarski Paradox:** How is it possible to take a ball, rip it apart, and assemble two balls of the same size?
- **Hilbert's Grand Hotel:** Can you always find room for any number of guests at a hotel if you have infinitely many rooms, even if all the rooms are taken?
- **St. Petersburg Paradox:** Should you play a game that costs you more than you would expect to win by playing it?
- **Simpsons Paradox:** How can a salesperson that sells less than another salesperson have a better sales record?

Simple Yet Unsolved Conjectures

A small note written on the edge of a lost notebook, an enigma emerging from a doodle, a devastatingly easy to understand pattern, what do these have in common? They are all the start of a story—a start to a question. How does the story end? No one knows yet; it's still being written.

- **Fermat's Last Theorem:** How long would it take to solve a problem scribbled on the edge of a sheet of paper when the author promised that he had a shockingly simple solution? You might want to watch this documentary: https://www.dailymotion.com/video/x223gx8
- **Collatz Conjecture/Syracuse Problem:** This devilishly simple looking problem can be explained to an elementary school student but has eluded mathematicians for nearly a century. It was said by one of the greatest mathematicians of this century, Paul Erdős, "Mathematics is not yet ready for such problems." What is this problem? What is known about it? What kinds of graphs/pictures come out of it?
- **Gilbreath's Conjecture:** While doodling on a napkin, Norman Gilbreath discovered a simple pattern involving prime numbers that has eluded mathematicians ever since. What is this pattern? Why is it not surprising that this conjecture has gone unsolved?
- **Goldbach Conjecture:** Another problem that is so simple that it can be explained even to an elementary school student. However, this problem has gone unsolved for over 270 years. It has even had a fictional story written about it—*Uncle Petros and Goldbach's Conjecture*—and been made into a movie—*The Calculus of Love*!
- **Millennium Problems:** Want to make a million dollars without ever leaving bed? Simply get up and on a sheet of paper you had next to the bed write down a proof of one of these problems! What are they? Have any been solved? How do they relate to Hilbert's 23 Problems? For that matter, who the heck is David Hilbert or this Landon Clay fellow?

Mathematical Figures

There is a misconception that mathematicians are quiet and aloof. People imagine mathematicians as a disheveled creature sloughing through basements or hiding among clutter in a busy office or perhaps staring at a chalkboard. While this is true in cases, the typical mathematician is a very social creature—working with others to solve problems. Moreover, a mathematician is a passionate creature. For a mathematician, Mathematics creates a passion for problem solving, a drive to learn and solve problems, and in some cases, murderous logic.

- **Andrew Wiles:** What would it be like to solve a problem that had gone unsolved for hundreds of years? It would certainly be the triumph of your career. But what happens to you after? Could this gift to the world turn into a curse? Try watching this documentary https://www.dailymotion.com/video/x223gx8 What is Wiles' emotional response in the beginning? Why do you think this is?
- **Bourbaki:** People imagine Mathematics as a very logical structure that all people agree on. However, this was not always the case. Paralleling Rand's John Galt, one should ask, "Who is Nicolas Bourbaki?"
- **Einstein & Gödel:** A mathematician and a physicist can make a great pair for a friendship but what happens when one dies?

- **Newton vs. Leibniz:** We often hear Newton created Calculus. But what happens in Mathematics when two different persons, separated by a continent, claim to have made the mathematical breakthrough of the century? Who gets the credit? How far would a man go to have his name go down in history? One might want to read Jason Bardi's *The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time*.

- **John Nash:** The life of a genius seems a charmed life. But what happens when the mind that once gave imagination and inspiration starts to work against you? Can you fight a madness from within? One might want to watch this documentary about John Nash https://www.youtube.com/watch?v=IxPFmvgPbos or perhaps this documentary https://www.youtube.com/watch?v=-HssdrQvTn4 One could also watch the somewhat fictional film about the life of John Nash: *A Beautiful Mind*. Where does the film differ from reality?

- **Stephen Hawking:** What would it be like to begin to make predictions about the nature of the universe but never be able to write them down? What would you do if you were told you were going to die in your 20's? One might want to watch *The Theory of Everything*. Where does the film differ from reality? What did Stephen Hawking do in Physics? Where there mathematical barriers?

- **Leonard Euler:** Could you be one of the most prolific mathematicians of all time while being blind? Who was Euler? What did he do? What things are named after him? Did we see any of his mathematics in this class? Try watching this *amazing* talk about his life and work: https://www.youtube.com/watch?v=fEWj93XjON0&index=18&list=PLybwJMJQaRE4G17CWduZD7OZOWAbelZdK

- **Alan Turing:** How do you break a code that everyone says cannot be broken to stop the biggest war in human history? How do you tell the difference between a computer and a person? What do these questions have to do with each other? What happened to Alan Turing?

- **Evariste Galois:** Would you die for the woman you love?

- **Paul Erdős:** How could a nomadic, drug taking, mathematician become one of the most published mathematicians of all time?

- **Alexandre Grothendieck:** A child of revolutionaries, Alexandre Grothendieck would follow in their footsteps by changing the way we approach deep mathematical problems. However, he was a true revolutionary. He often wrote his mathematics blended with personal philosophy. Eventually, his despise with the politics of mathematicians caused him to burn thousands of pages of unpublished manuscripts and disappear. Who was this mysterious figure? Why was he so passionate? What was he passionate about? What did he do to be called the "greatest mathematician of the 20th century"?

- **Professor Petryshyn:** What happened when in a series of events beginning when a professor discovered an error in his textbook?

- **Ted Streleski:** Imagine you spent 20 years trying to earn your Ph.D. to no avail. What lengths would you go to avenge perceived wrongdoings?

- **Ted Kaczynski:** This man terrified a nation. What name was he better known by?

- **André Bloch:** What would you do if Mathematics "told you" to kill?

- **Hardy, Littlewood, & Ramanajuan:** What would you do if you received a letter from a person who taught themselves so much mathematics that their ability rivaled—even surpassed—your own?
- **Grisha Perelman:** What could be more mysterious than a man that solves a hundred year old problem, refuses the highest honor in Mathematics, turns down $1,000,000, then quits Mathematics without a word?

Mathematical Oddities

Often times, Mathematics tells us something surprising about our world and the reality in which we exist. Furthermore, Mathematics tells us about all possible worlds of which we can conceive. These results are counterintuitive or seem outright contradictory. Nevertheless, if we accept the axiomatic system we used and its validity to represent the situation at hand, their truth value is absolute.

- **Turning a Sphere inside Out:** Can you take a ball and turn it inside out without breaking it? A good place to start would be YouTube. By the way, who is Bill Thurston?
- **Rope Around the Earth:** Imagine taking a huge rope and wrapping it around the Earth so that the ends touched and the rope is taunt against the ground. How much more rope would be needed so the rope could be lifted over your head with the ends still touching? The answer might surprise you!
- **Mind-bending Creations:** Can you make a sheet of paper have only one side? What does a bottle that pours out water into itself look like? What does a 4-dimensional cube look like? Investigate the Möbius strip, Klein bottle, tesseract, etc.
- **False Proofs:** There is an internet phenomena of showing that Mathematics is "incoherent" or "useless" because it is contradictory via proofs of such things as "proving" $0 = 1$. What are some of these proofs and where do they go wrong? Have there been proofs in Mathematics of repute which were later proved to be wrong? What is the efficacy of Mathematics?
- **"Knowing" in Mathematics:** If you prove something in Mathematics, is it always true? When are things true and when are things false? Is there an answer for every question in Mathematics? Is Mathematics even "true" or are we "making it up"? It might be fun to watch this talk before beginning: https://www.ted.com/talks/eduardo_saenz_de_cabezon_math_is_forever?language=en How does this relate to Gödel's Incompleteness Theorem?
- **Mathematics of Synchrony:** How do you explain how or why fish swim together or why birds flock? Can non living things also "join forces"? Why do we see symmetry and synchrony spontaneously evolve in nature? You might want to start by watching: http://www.ted.com/talks/steven_strogatz_on_sync?language=en
- **Solving a Polynomial:** We can always solve the question $ax + b = 0$ for x. The quadratic formula lets us solve $ax^2 + bx + c = 0$ for x. How about the cubic equation, $ax^3 + bx^2 + cx + d = 0$, can we always solve this? How about the quartic equation? What about a general polynomial of degree n? What polynomials can be solved? What does this have to do with a teenager named Evariste Galois?

- **Mathematics of Calculations:** How do you do find 24×37? How about $\sqrt{287}$ or $\sqrt[3]{2\pi^4}$ or solve $3x^4 - \sqrt[5]{2x} - \sin\ln(x) = 0$? How does a calculator do these things so quickly? How would you design such an object or even tell a person how to do it?
- **Distribution of Prime Numbers:** Recall the prime numbers: $2, 3, 5, 7, 11, 13, 17, \ldots$. Why are these the prime numbers? Why are they so close together at first, then spread apart, then close, et cetera? What is the pattern to the primes? What Mathematics has trying to answer this question led to? Who knows, it might be fun to start by watching: http://www.ted.com/talks/adam_spencer_why_i_fell_in_love_with_monster_prime_numbers?language=en
- **Folding:** How many times can you fold a sheet of paper? How tall would this sheet be if you folded it 20 times? Who set the world record for folding?

Numbers

Mathematics began with the problem of counting. Its early years were spent observing and explaining patterns in numbers. The beauty and absoluteness of this structure created a mythicism and religion around numbers. What is the past and future of numbers today? What kind of life have numbers taken on today?

- **Numerology:** Numbers were once believed to hold special meaning and held certain powers. This led to the superstition of today around numbers such as 7, 13, 666, et cetera. What are examples of these numbers? What did/do they mean? What societies had these number mythicism? How did they arise?
- **Birth of Zero:** It took many cultures years and great difficulty to create the number zero. What culture created zero first? What obstacles were there to overcome? Did zero or negative numbers come first?
- **Amicable Numbers:** What does it mean for two numbers to be amicable? Who was the first to find these numbers? What are examples of these numbers? You might want to watch the talk on Euler given above.
- **Monster Group:** What is the monster group? How did it get its name? What does it have to do with the real world?
- **Skewes' Number:** How large is this number? Is it actually useful?
- **Graham's Number & TREE(3):** What is Graham's number? What is TREE(3)? How big are these numbers? You might want to type these into YouTube to see what videos you might find.
- **Normal Number:** Can a number predict the future of all mankind? Has such a number ever been found?
- **Golden Ratio, $\pi, \mathbf{e}, \mathbf{i}$:** What are these numbers? What do they have to do with nature? Are there ways to picture these numbers? When were they discovered? Who discovered them? Are there relations between these numbers?
- **Transfinite Numbers & Conway Numbers:** We know that ∞ is not a number but can it be made into a number? Is there a number system where $\infty + \infty = 2\infty$ makes sense? What is the mathematics of infinity?

- **Data Mining:** With the advent of computers, we are now more than ever able to collect and analyze large amounts of data. This has led to deep connections and models never before thought possible. What kind of forms can these numbers take that they never took before? You might want to watch http://www.ted.com/talks/david_mccandless_the_beauty_of_data_visualization?language=en#t-861094 or perhaps http://www.ted.com/talks/aaron_koblin?language=en#t-899877 How about the man that can "feel" numbers: https://www.youtube.com/watch?v=Kf3-el-dJAw

Fractals Imagine an object which looks the same, no matter how close you are to it. You back away only to realize that the object still looks exactly the same! How can you create an object that is "infinitely self similar"? Could this be done by an elementary school student? Can objects of such symmetry even exist in nature? What does this all have to do with the coastline of Britain?

- **Mandelbrot Fractal:** What is this well known design and how is it generated?
- **Buddhabrot Fractal:** Could a series of numbers generate a religious icon?
- **Menger Sponge:** How do you generate this seemingly simple figure?
- **Koch Snowflake:** Can you create a formula which generates a snowflake? How about a fern? What kinds of objects in nature can be modeled with fractals?
- **Science Applications:** Where do fractals appear in the sciences? How can a better understanding of fractals improve technology, computer simulations, or mathematical models of the world?
- **Artistic Applications:** What art is created using fractals? Can you use a fractal to create any image? Where do fractals appear in designs for buildings or logos?

Mathematical Art & Design Many famous works of art have intricate mathematical designs hidden within. Modern architecture is characterized by a deep symbiosis between art, design, and mathematics. In fact, some artists create by starting with a mathematical concept in mind—such as M.C. Escher. At the heart of it all, is there a difference between Mathematics and Art?

- **Spherical Geometry:** What do flat objects look like on a sphere? How does this relate to maps? What does photography look like when the photo is put on a sphere rather than normal photograph paper?
- **Non-euclidean Geometry:** What do things look like in a world where you can walk up a flight of stairs and actually end up downstairs? How is this type of space related to the works of M.C. Escher?
- **Tiling Patterns:** How do you fill a paper with squares with no space wasted? With octagons? Can you do this with spheres? What is the most efficient way of doing this? Does this have any relation to "real life"? What makes Non-euclidean Geometry different from the Euclidean Geometry that "everyone" knows? You may want to also Google "space filling curve."

- **Wallpaper Patterns:** What is the symmetry in different art forms? What fields of Mathematics does this relate to? Does this allow us how to tile a floor easily with beautiful repeating patterns? Even better, what does this have to do with the "birds and the bees"?
- **M.C. Escher:** Who is M.C. Escher? What type of art did he create? What is the mathematics of his art? Would the every day person recognize any of this artwork?
- **Origami:** What is the mathematics of origami? How can we use this mathematics to design previously unimaginable designs? Can origami let us see distant galaxies, develop easily transportable products, or even save a life? Find out more in this TED Talk: http://www.ted.com/talks/robert_lang_folds_way_new_origami?language=en
- **Mathematical Designs:** Can you make a sheet of paper have only one side? What does a bottle that pours out water into itself look like? What does a 4-dimensional cube look like?
- **XKCD:** How can you love of Mathematics and Science along with doodling turn into a living? Check out https://xkcd.com/ and https://what-if.xkcd.com/.

Mathematics & the Humanities

Mathematics is the science of rational thoughts and patterns. So how could Mathematics ever accurately reproduce and predict the inner workings of humanity? Can Mathematics explain concepts such as love or friendship? What does Mathematics say about humanity and its future?

- **Mathematics & Language:** Can Mathematics be used to explain why history has progressed the way it has? Can it make predictions about the future of language? Take a peek in this TED Talk https://www.ted.com/talks/jean_baptiste_michel_the_mathematics_of_history.
- **Math & Love:** Is there a deep math behind how we love? Can an algorithm determine who is the right match for you? Try watching http://www.ted.com/talks/hannah_fry_the_mathematics_of_love Is the Mathematics here real or are these just truisms? Is Math able to or will it be able to answer these questions? What might this have to do with advertisement tracking in applications such as Facebook?
- **Math & Crime:** How are statistics used in police work or trials? Are the statistical conclusions always valid? What is "blinding by science"? You might want to check out Mlodinow's "The Drunkard's Walk: How Randomness Rules Our Lives." It might be interesting to look at Simpson's Paradox.
- **Math & War:** Is mathematical progress always a good thing? What type of harm can Mathematics do in the world? Who is G.H. Hardy and what did he apologize for? What did he wish for and did he get his wish?

Mathematics & Sports

How do you know who is the best player in any given sport? How do you design an automobile to be as aerodynamic as possible, a luge with as little friction as possible, or shoes that are supportive and weigh as little as possible?

- **Sabermetrics:** Can baseball alone create a whole field of Mathematics?
- **Sports Design:** How do mathematicians design sports equipment to give athletes the greatest possible chance at victory?
- **"Game" Sports:** How would you advise a Scrabble athlete how to win? How about a competitive chess player? How about a competitive trivia player? Could a mathematician and computer scientist work together to create a machine which beats a human every time? What is Deep Blue and how does this connect to the computer Watson? You might want to watch: https://www.ted.com/talks/ken_jennings_watson_jeopardy_and_me_the_obsolete_know_it_all?language=en or perhaps even https://www.youtube.com/watch?v=4svcCJJ6ciw
- **Theorycraft:** How do you design computers to play against humans in games that involve many layers of strategy? How does this challenge what we consider sports or what we consider thought versus blind rule obedience?
- **Game Theory:** How does Game Theory relate to sports? What kinds of games does it apply to? When should one apply these rules? When do these rules work and when do they fall apart?

Mathematics & Computers

Computers are full of mathematics as they are quite literally built using mathematics. However, when one applies advanced mathematical concepts with the computation power of the modern machine, the sum of whole is greater than its parts. What can computer algorithms and programs do for humanity? Can we actually model the universe or see human thought?

- **Turing Test:** If you are talking to someone through a computer, how do you know that person is a human? How could you tell if it was a computer? What does this tell you about what it means to be human; what it means to be intelligent; what it means to be alive; what it means even to exist?
- **GPS:** How does an object tens of thousands of miles away determine where you are within fractions of an inch?
- **RSA:** How do you share a secret message with someone when you both do not trust each other? Can two people share encoded information without either knowing the code?
- **Data Mining:** With the advent of computers, we are now more than ever able to collect and analyze large amounts of data. This has led to deep connections and models never before thought possible. Can how we physically look at this data lead to all these breakthroughs? Can this do something for humanity beyond better real world models? You might want to watch http://www.ted.com/talks/david_mccandless_the_beauty_of_data_visualization?language=en#t-861094 or perhaps http://www.ted.com/talks/aaron_koblin?language=en#t-899877

- **Fuzzy Logic:** Which shade of blue is most "blue"? What shirt should you wear today? Is it true that "all" birds fly? What is the Mathematics of "probably" logic?
- **Biological Engineering:** Can computers and the mathematics of folding revolutionize how we can engineer for health? What can playing a game do for mankind? You might want to read this article: http://cosmiclog.nbcnews.com/_news/2011/09/16/7802623-gamers-solve-molecular-
- **Fraud Detection:** How can you tell if a finance report is falsified? How can you determine who is at the heart of a conspiracy? What is the mathematics of the hidden? You may want to watch/read about how videos of famous persons can now be faked.
- **Animation:** How do you design computer models of places and things that look real? What is surface interpolation? What are quaternions? How do these problems relate to the sciences? You might want to watch https://www.youtube.com/watch?v=mX0NB9IyYpU
- **Shopping:** What do the barcodes on products mean? How do you manage shipping thousands of products to millions of individuals? Do humans even do any of this work anymore? You might want to watch this: https://www.youtube.com/watch?v=qfeymoF8pb4
- **Stenography:** Can those trolling internet cat photos actually be hiding an insidious message? Reading the first few pages of this http://niels.xtdnet.nl/papers/practical.pdf might give you a place to start.
- **Mind Reading:** Is it possible to use electronic waves to read the human mind? You might want to watch http://www.ted.com/talks/mary_lou_jepsen_could_future_devices_read_images_from_our_brains?language=en#t-379348 or this talk http://www.ted.com/talks/tan_le_a_headset_that_reads_your_brainwaves?language=en#t-578691 or maybe even this talk http://www.ted.com/talks/henry_markram_supercomputing_the_brain_s_secrets
- **Advertising and Searching:** Google's search algorithm is a billion dollar algorithm. How companies choose advertisements which target you? How do you find out if two people might know each other or determine how related two words are? You might start by watching https://www.youtube.com/watch?v=v7n7wZhHJj8
- **Wolfram Demonstrations:** What happens when Math collides with open programming? Check out http://demonstrations.wolfram.com/