Rates Project

Instructions

Read the motivational section to get an idea of the purpose of this project and get a small amount of background information needed to do this project. Then work through the concept problems. Do these problems in the order that they are written. These concept problems build on each other to take you from the most basic rate problems to the more advanced introductory problems. Neatly write up these problems to submit. Each problem should appear on its own sheet of paper (if it only appears on one side of a page, that is fine). Be sure to include units throughout your calculations for these problems. You will have to look up and estimate values yourself to solve these problems—as is often the case in real world calculations. For example, a question might ask how long it takes for a car to accelerate to 15 mph. But this depends on the type of car and how fast cars typically accelerate. So you will have to assume a type of car or look up possible acceleration values for cars generally and make a reasonable estimation/assumption of your own.

Once you complete the concept problems, proceed to “The Problem" section. Here you will find an open ended problem to explore using the concept of rates. Try to answer the question using the mathematics you have learned and be as detailed as you can. Write up your solution to this problem. This write up should consist of the following parts:

1. Background: Give some background to the question. What were you looking to do? Briefly state what were the results and how you came to them.
2. Assumptions: What assumptions did you make about the problem in order to do it? Do they change throughout the problem? What type of mathematics did you use?
3. Calculations: Show the computations you used to solve the problem. Be sure to write these up neatly and use mathematical notation. Do not forget to include units throughout any calculations. You should include some error calculations.
4. Conclusion: What were your results? Did they seem valid? Discuss your sources of error and how you overcame or included them. How could you perform a more detailed analysis? What, if anything, did you learn about the problem, the mathematics you are learning, or the world from this problem?

Do not panic if you cannot come to precise mathematical conclusions or your models are not very advanced. This project is about applying your mathematics to the “real world.” Even if you cannot make any computational mathematical progress towards the problem, you can still receive high marks! Write down your thoughts about the problem, what you tried, what worked and what did not work, what your logic or common sense should tell you should be true or false about the problem, evaluate your conclusions or how you might have gone about the problem, et cetera. Good analyses and discussions can potentially receive as much credit as a good mathematical model.

You may work with others or receive help with anyone you would like for this project, especially from your instructor. Real mathematicians and scientists consult often with others! However as with real mathematicians or scientists, credit needs to be given where credit is due. Use footnotes or other indication in the text or calculation where you obtained any values, where or from whom you received help, et cetera. Cheating or plagiarism will not be tolerated and can result in a zero for the project. Indeed, where you get your information could be a valuable part of your error analysis or discussion sections!

Grading

The project must be submitted by the start of class Thursday July 23, 2015. Your project will receive its grade based on the following scale.

Unsatisfactory: 0	No problem turned in, an illegible project is turned in, or the project is virtually unintelligible.
Good: 1–3	• Some effort was shown. • Units were used infrequently or not at all. • Parts were missing or incomplete. • There was little or no discussion about the problems. • The project was not organized or poorly written.
Great: 4–6	• Good effort was shown in completing the project • Use of units was occasionally missing. • No parts of the project were missing or incomplete. • All project sections were discussed and given proper treatment. • The project was organized and neatly written.
Excellent: 7–9	• Significant effort was demonstrated. • Units were used properly throughout. • No parts were missing or incomplete. • The project sections showed significant and thoughtful discussion and analyses. • The project was very well organized and neatly written.

Motivation

In this project, you will investigate the relationships between distance, velocity, and time. This project will make extensive use of one of the most useful formulas/concepts in all of *school* Mathematics: $d = vt$; that is, distance is the product of the velocity with time. This concept is used throughout high school and introductory college Mathematics as well as many general exams, i.e. the SAT and the GRE. This concept can be stated generally as

$$\text{Total Amount (of Change)} = \text{ Rate of Change} \times \text{ Time}$$

In the case of velocity of distance, your total change is your (change in) distance, d, and your rate of change is your velocity, v. But notice that this concept is more general as the situation could have been just about anything. As a few examples

$$\begin{aligned} \text{Liters Chemical Added } &= \text{ Rate You Pour Chemical In } \times \text{ Time} \\ \text{Change in Velocity } &= \text{ Acceleration } \times \text{ Time} \\ \text{Money Made } &= \text{ Hourly Pay } \times \text{ Time} \\ \text{Pages Typed } &= \text{ Pages Per Hour } \times \text{ Time} \end{aligned}$$

Of course, notice that this only calculates a total change. So generally, we have

$$\text{Total } = \text{ Initial Amount } + \text{ Change } = \text{ Initial Amount } + \text{ Rate } \times \text{ Time}$$

So in the case of distance and time, $d = d_0 + vt$, where d_0 is your initial distance traveled. In the case of velocity and time, $v = v_0 + at$, where v_0 is your initial velocity and a is your acceleration. Notice these are all examples of linear functions. Observe that distance is linear in time: $d = vt$. Here your average rate of change, i.e. slope, is the velocity.

In most contexts, the casual $d = vt$ is written. But what is meant is $d(v) = vt$. This is because the distance you travel depends on your velocity—you would travel different distances if you traveled at 64 mph for a fixed time rather than constantly accelerating/decelerating. Notice that if the velocity is not constant, $d(t)$ is not a linear function because the velocity v is not always the same for different times t. In this case, d is now a function of two variables: $d(v, t) = d_0 + vt$. Of course, one could have regarded it as a function of two variables originally $d(v_0, t) = d_0 + v_0 t$, where one always chose the same fixed velocity: v_0.

Concept Problems

Problem 1: An ICBM races into the atmosphere. How many seconds does it take an ICBM to exit the Earths atmosphere?

Things to consider:

(a) How does this relate to $d = vt$?

(b) How fast does an ICBM travel?

(c) For far do you have to go to reach space?

Problem 2: A satellite rotating about the Earth travels approximately in a straight line when viewed over a short period of time. Consider a satellite 640 km above the surface of the Earth. It covers about 905 km in two minutes. Find the average velocity of the satellite in m/s. Does the satellite have to travel at this speed at any point? Explain. Does this model work if we consider long periods of time? Why does it work or could it be made to work? Explain.

Things to consider:

(a) What is the equation for average velocity?

(b) What if it always traveled faster or always slower than its average velocity?

(c) What does the orbit of the satellite look like? How is this different than the typical 'shape' of a path you would use $d = vt$ for? How might this affect things? Why?

Problem 3: On the first manned trip to Jupiter's moon Europa, astronauts use the moon's gravity to slingshot them towards Europa at 33 km/s. They then turn on their booster engines and accelerate at 3 km/s^2 for 20 seconds and then turn on a small auxiliary engine to accelerate at 12 m/s^2 for 2 hours 20 min. How fast are the explorers hurtling towards Europa now? Approximate as roughly as you can the time to Europa for the explorers at their current speed.

Things to consider:

(a) How far is it from the Earth to Europa? Is this distance fixed?

(b) How does the acceleration affect the velocity? How does the velocity affect the distance traveled? How do you combine these affects?

(c) How can you use $d = vt$ for the final calculation?

Problem 4: A 120-$A/2$ production robot finishes production of its computer chips in 46 hours. A new model, 341-B, is introduced and finishes production of the same chip in 38 hours. The company uses both robots to engineer the chip until the 120-$A/2$ model breaks and will be replaced by the new model. How long does it take the robots working in tandem to finish the production? What is the time saved in production compared to the original situation? What is the percent increase in production time?

Things to consider:

(a) How does this relate to $d = vt$?

(b) Do this for a simple case: you work at rate x and your friend works at rate $2x$, how long does it take you to finish a task that before took you an hour?

Problem 5: A genetically engineered bacteria produces a protein for medicine production. A collection of the bacteria, strain A, produces a single dose in 6 days. A second batch, strain B, produces a dose in 8 days. A third bath of bacteria, strain C, is being engineered to help speed production. How fast will this newly engineered bacteria need to produce a dose if the combined collections of bacteria strains are to produce a dose in merely 2 days? Suppose that strain B dies and strain A has been producing a dosage for 1.5 days. But then strain C is finally engineered and begins producing a dose. How long will it take strain A and C working together to finish producing the dose?

Things to consider:

(a) Relate this to the previous problem.

The Problem

You are driving in the right lane on a highway. You look in the mirror and notice a car in the left lane approaching you to pass you. Determine the speed of the other car.

Some things you might want to consider—though certainly not a comprehensive list—thinking about or including in your analyses:

1. Does the speed of your car or the speed of the other car matter?
2. Does it depend on the type of car?
3. Can you pretend that one car is not even moving?
4. Does the fact that you were looking in the mirror matter?
5. Does the direction of the road matter?
6. Does it matter if either of the cars is accelerating?
7. You may not know their speed, but you know yours. What does this have to do with how long it takes them to pass you?
8. What would happen if there were other cars on the highway? Could you calculate your speed or theirs watching a car pass another car instead of your own?
9. Does this situation extend to other types of vehicles such as sea or air vehicles? What if they were space vehicles? Would any additional assumptions have to be made?
10. Are there forces you are ignoring? How might they effect the situation?
11. Explain how this might relate to speed radars.