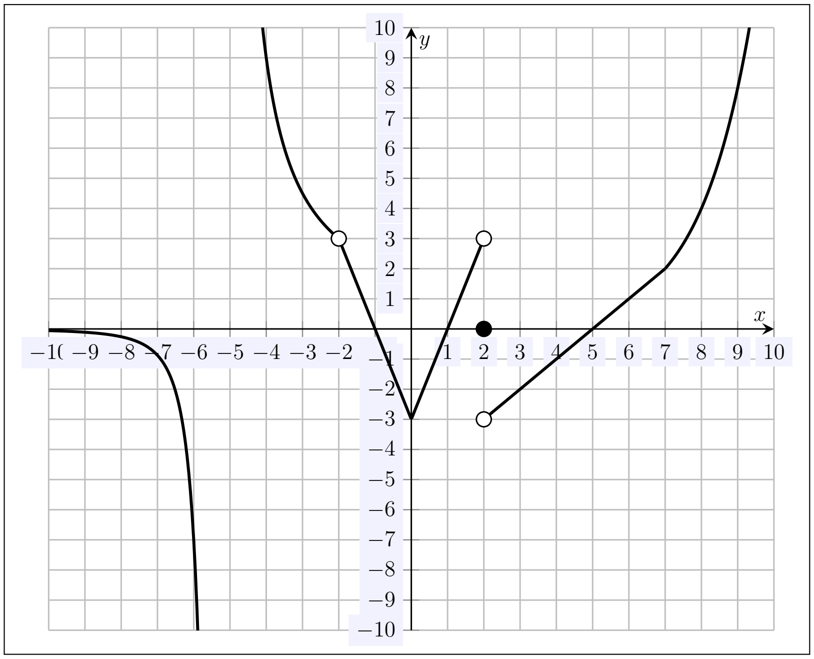
Math 295: Exam 1 **Name:** *Caleb M^c^Whorter — Solutions*
Fall – 2016
09/23/2016
55 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 7 pages (including this cover page) and 7 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
Total:	0	

1. Mark the following statements True or False:

	Statement	True	False
(a)	If a function is differentiable, then it is continuous.	✓	☐
(b)	If $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ exist, then $\lim_{x\to a} f(x)$ exists.	☐	✓
(c)	If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ exist.	✓	☐
(d)	If a function is continuous, then it is differentiable.	☐	✓
(e)	$\lim_{x\to a} (f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.	☐	✓
(f)	Polynomials are everywhere continuous.	✓	☐



2. Use the plot of the function $f(x)$ above to answer the following questions:

(a) $f(2) = 0$

(b) $\lim_{x\to 2^-} f(x) = 3$

(c) $\lim_{x\to 2^+} f(x) = -3$

(d) $\lim_{x\to 2} f(x) = \mathit{DNE}$

(e) $\lim_{x\to -2^-} f(x) = 3$

(f) $\lim_{x\to -2^+} f(x) = 3$

(g) $\lim_{x\to -2} f(x) = 3$

(h) $\lim_{x\to -\infty} f(x) = 0$

(i) $\lim_{x\to \infty} f(x) = \infty$

(j) What is the y-intercept of $f(x)$?
$(0, -3)$

(k) What are the zeros of $f(x)$?
$x = -1, 1, 2, 5$

(l) If $f(x)$ has any vertical asymptotes, give their equation.
$x = -5$

(m) Where is $f(x)$ continuous?
Everywhere on $\mathbb{R}$ *except* $x = -5, -2, 3$

(n) List at least 4 values for x at which $f(x)$ is not differentiable.
$x = -5, -2, 0, 2, 7$

3. Calculate the following limits. Be sure to show all work necessary to your computation!

$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-x-6}$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-x-6} = \lim_{x\to 3} \frac{(x+1)(x-3)}{(x+2)(x-3)} = \lim_{x\to 3} \frac{x+1}{x+2} = \frac{4}{5}$$

$\lim_{x\to 0} \frac{3x}{\sin 5x}$

$$\lim_{x\to 0} \frac{3x}{\sin 5x} = 3\lim_{x\to 0} \frac{x}{\sin 5x} \cdot \frac{5}{5} = \frac{3}{5} \lim_{x\to 0} \frac{5x}{\sin 5x} = \frac{3}{5} \cdot 1 = \frac{3}{5}$$

$\lim_{x\to 0} \left(\csc x - \cot x \right)$

$$\lim_{x\to 0} \left(\csc x - \cot x \right) = \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) = \lim_{x\to 0} \frac{1-\cos x}{\sin x} \cdot \frac{x}{x} = \lim_{x\to 0} \frac{x}{\sin x} \cdot \frac{1-\cos x}{x} = 1 \cdot 0 = 0$$

4. Evaluate the following limits. You do not need to justify your answer.

   (a) $\lim_{x\to\infty} \frac{5x^2+8x-4}{3x^3-4x^2-x+6} = 0$

   (b) $\lim_{x\to\infty} \ln(2x^2+x+1) - \ln(7x^2-x-1) = \ln(2/7)$

   (c) $\lim_{x\to\infty} \frac{2^x \sin x}{x^2+2x+1} = \infty$

   (d) $\lim_{x\to-\infty} \frac{2x+3}{\sqrt{4-x}} = -\infty$

   (e) $\lim_{x\to\infty} x\sin(1/x) = 1$

   (f) $\lim_{x\to\infty} \frac{2x^4+6x-4}{3x^2+4x+2} = \infty$

   (g) $\lim_{x\to\infty} \frac{\ln x}{x+1} = 0$

   (h) $\lim_{x\to\infty} \frac{3x^2+4x-1}{4x^2-5x-2} = \frac{3}{4}$

5. Use the Squeeze Theorem to show $\lim_{x \to 0} x^2 e^{\sin(1/x)} = 0$.

We know that $-1 \leq \sin(1/x) \leq 1$. *But* e^x *is monotone increasing so that we have*

$$\begin{aligned} e^{-1} \leq & \quad e^{\sin(1/x)} \leq \quad e^1 \\ \frac{1}{e} \leq & \quad e^{\sin(1/x)} \leq \quad e \\ \frac{x^2}{e} \leq & \quad x^2 e^{\sin(1/x)} \leq \quad ex^2 \end{aligned}$$

But $\lim_{x \to 0} \frac{x^2}{e} = 0$ *and* $\lim_{x \to 0} ex^2 = 0$. *Therefore by the Squeeze Theorem,* $\lim_{x \to 0} x^2 e^{\sin(1/x)} = 0$.

6. Use the Intermediate Value Theorem to show that the equation $e^x = x^2 - 1$ has a solution in $[-2, 0]$.

Notice that $e^x = x^2 - 1$ *has a solution on* $[-2, 0]$ *if and only if* $e^x - x^2 + 1 = 0$ *has a solution on* $[-2, 0]$ *if and only if* $f(x) = e^x - x^2 + 1$ *has a root in* $[-2, 0]$. *Observe that* $f(x)$ *is a sum of continuous functions and is therefore continuous. Notice also*

$$\begin{aligned} f(0) &= e^0 - 0^2 + 1 = 1 - 0 + 1 = 2 > 0 \\ f(-2) &= e^{-2} - (-2)^2 + 1 = \frac{1}{e^2} - 4 + 1 = \frac{1}{e^2} - 3 < 0 \end{aligned}$$

Therefore by the Intermediate Value Theorem, there is a $c \in [-2, 0]$ *so that* $f(c) = 0$. *But then* $f(c) = e^c - c^2 + 1 = 0$. *Therefore,* $e^c = c^2 - 1$, *and thus* $e^x =^2 -1$ *has a solution in* $[-2, 0]$.

7. Use the definition of the derivative to find the derivative of $f(x) = x^2 - 3x + 1$.

$$f'(x) = \lim_{x\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{((x+h)^2 - 3(x+h) + 1) - (x^2 - 3x + 1)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h + 1 - x^2 + 3x - 1}{h}$$

$$= \lim_{h\to 0} \frac{2xh - 3h + h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2x - 3 + h)}{h}$$

$$= \lim_{h\to 0} (2x - 3 + h)$$

$$= 2x - 3$$