Math 295: Exam 3 **Name:** ______________________________
Fall – 2016
11/18/2016
55 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 7 pages (including this cover page) and 5 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	15	
2	15	
3	10	
4	10	
5	10	
Total:	60	

1. (15 points) Compute each of the following limits. Be sure to justify your answer completely.

   (a) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

   (b) $\lim_{x \to \infty} \frac{x^2 + 3x + 1}{e^{2x} - x + 2}$

   (c) $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$

(c) $\lim_{x \to 0^+} x^2 \ln x$

(d) $\lim_{x \to 0} (\cos x)^{3/x^2}$

2. (15 points) Let $f(x) = \dfrac{x^2 - 2x + 4}{x - 2}$.

(a) Find and classify any critical values for $f(x)$.

(b) Find the intervals on which $f(x)$ is increasing and decreasing.

(c) Find the intervals on which $f(x)$ is concave and convex.

(d) Find the local and absolute maxima/minima on $[0, 6]$.

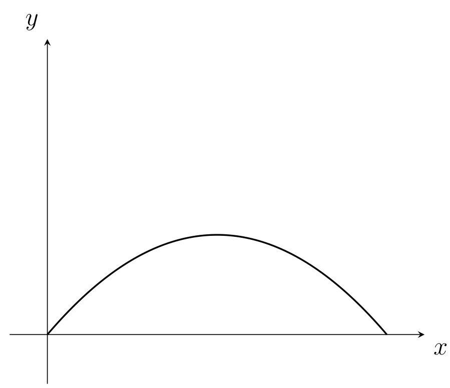
3. (10 points) Complete the following parts:

(a) If $0 < a < b$, show that

$$\sqrt{b} - \sqrt{a} < \frac{b-a}{2\sqrt{a}}$$

(b) Is there a differentiable function on $[-1, 2]$ such that $f(-1) = -2$, $f(2) = 5$, and $f'(x) < 2$ for all $x \in [-1, 2]$? Justify your answer completely.

4. (10 points) If a projectile is fired at an initial velocity of v_0 at an angle θ—neglecting air resistance—the projectile follows a parabolic path. A simple calculation shows that at time t, the x-coordinate of the projectile is given by $x(t) = tv_0 \cos\theta$ and the y-coordinate is given by $y(t) = tv_0 \sin\theta - gt^2/2$, where g is the acceleration due to gravity.



(a) Solve for t in $x(t)$ and use this to find y in terms of x.

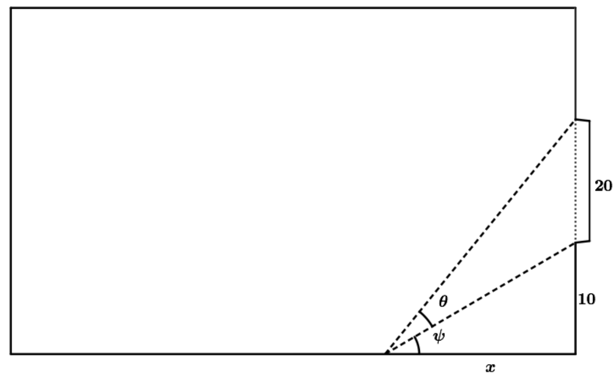
(b) The range of a projectile is the distance traveled before impacting the ground. Explain why setting $y = 0$ allows you to find the range of the projectile.

(c) To find the range, set $y = 0$ and find a nonzero solution for x. Call this value R for the range and hence show $R(\theta) = \dfrac{v_0^2}{g}\sin(2\theta)$. [You will need $\sin 2\theta = 2\sin\theta\cos\theta$.]

(d) Find the angle θ that maximizes the range R. Be sure to justify why this is a maximum.

5. (10 points) Investigate the following problem by completing the parts below. Be sure to justify all steps.

Imagine a soccer player standing on the sideline planning to try to kick the ball into the goal. If the player stands in the bottom right corner of the field, no angle they can kick the ball at will score a goal (ignoring spin). On the other hand, if they stand in the bottom left corner of the field, they have a narrow angle window in which to kick the ball so by the time the ball "arrives" at the other side, it will land in the goal. There then must be a spot along the bottom sideline that gives the "best" possible shot in terms of the largest possible angle to kick the ball and score a goal. Here you derive this spot along the bottom of the field.



(a) Show that $\psi = \arctan\left(\frac{10}{x}\right)$.

(b) Show that $\theta + \psi = \arctan\left(\frac{30}{x}\right)$.

(c) Use the previous two parts to find the distance from the corner x that maximizes the shot angle θ. [You *do not* need to use a derivative test to prove it is a maximum.]

(d) For this distance x, what is the maximum possible angle shot, θ?